Admissible Designs of Debt-Equity Swaps for Distressed Firms: Analysis, Limits and Applications
Franck Moraux: Université De Rennes 1, Iae Rennes Et Crem, Rennes, France
Patrick Navatte: Université De Rennes 1, Iae Rennes Et Crem, Rennes, France

CONTACT: Franck Moraux, Université de Rennes 1, IAE Rennes et CREM, 11 rue Jean Macé, 35000 RENNES, France.
Email: franck.moraux@univ-rennes1.fr

Abstract

This paper reconsiders the design of debt-equity swaps that are common tools to financially restructure distressed firms. While an ad hoc approach consists in characterizing a set of three parameters, we demonstrate that a system of two equations defines admissible designs. Hence, assuming that creditors do not want to bankrupt the firm nor they want to evict completely current equity holders, we solve the debt holders’ design problem. We then undertake an in-depth analysis of corresponding solutions and we show that debt-equity swaps can significantly increase the probability of being reimbursed of the remaining due payment in the next future.

Keywords: Contingent Claim Analysis, Debt Equity Swap, Design.
Code JEL: G3, G33, G34.
Introduction

In situations of default, swapping debt for new equity is a possible method to reorganize financially distressed firms. Because debt holders become essentially equity holders, this solution can be viewed as a way to align interests of both parties in problematical contexts. As a matter of facts, empirical studies conclude that this approach is rather popular. Gilson and al. (1990), for instance, study 169 successful restructurings undertaken between 1978 and 1987. The authors report that equity securities were distributed to creditors in almost 75 percent of their sample. This result appears significant even if disparities seem to exist between loans and bonds. It is also worthy to note that a similar percentage of the restructuring results in a reduction of the debt payments. Few years later, James (1995) provides useful statistics on bank debt restructurings. He focuses on capital structures made of private and public debt; his sample covers 1981 through 1990. This author finds that 80% of public debt restructurings involve debt-equity swaps against about one in three for the private ones. He shows however that this difference is only an expected consequence of the conflicts between lenders. Interestingly, private lenders make concessions in terms of reducing principal in virtually all the cases (91%). In exchange, banks obtain a significant percentage of the firm’s

---

1 There are many ways to financially reorganize distressed firms. Among others, one finds, the debt forgiveness, the reduction of interest or principal and the extension of maturity. For more details on these, please consult Asquith et al. (1994), Gilson and al. (1990), Longstaff (1990), John (1990) and the references herein.

2 51.4 percent and 86.7 percent for respectively the private outstanding and the public debt.

3 More precisely, he finds that banks do not take equity without public bond holders also restructure their claim and that they don’t swap their debt for equity if bond holders swap their debt for debt.

4 The concession appears even rather significant since the average reduction in principal is 41% of the loan amount. Unfortunately James does not comment this figure in deeper details. We
common stock. They receive on average a little more than half of the equity when bondholders are present; one third when there is no public outstanding. This result on creditors’ participation in the equity confirms those of Gilson (1990) indicating that banks, on average, become the largest stock holder after the restructuring. Finally, James (1990) also finds that banks maintain a substantial stake years after the restructurings, although there exist various legal and regulatory factors that can constraint banks from stocks holdings.

Compared to this amount of evidences, the theoretical literature on debt-equity swaps appears very thin. This is rather surprising because this subject is of crucial importance for every party involved in private workouts. Previous research dealing on debt-equity swaps mainly focus on determinants and motives for such a reorganization scheme. None of them shares our objective to understand their design. James (1995), for instance, examines conditions under which lenders take equity in loan work outs. He questions factors influencing a bank lender’s incentives to scale down its debt unilaterally when the distressed firm has public debt outstanding too. Exploring determinants of debt-equity swaps only, he offers no specific insight on the way they should be designed. Isakawa (2002) relates swaps to the agency costs associated to the managerial behaviour. He defends that swapping debt for equity is an appropriate way to solve problems caused by managerial opportunism. In other theoretical works, debt-equity swaps play the role of a non trivial reorganization scheme. Fan and Sundaresan (2000) and Ericsson and Renault (2006), e.g., mobilize them as emblematic instruments to model out-of-court renegotiation.

Our paper contributes to the literature by studying the design of debt-equity swaps. We solve the debt holders’ design problem. We assume that they restructure their debt in order to avoid liquidation costs and that they choose

will defend in what follows that this reduction should not be arbitrarily chosen at all but rather closely related to parameters characterizing the debt-equity swaps.
the debt-equity swaps parameters so as to optimize their positions. We show that there is no straightforward solution and that the set of admissible designs requires solving simultaneously a system of two equations with three unknown variables. It appears that an extra condition is needed to find an optimal (non corner) solution. By assuming a given participation level in the new equity (lower than 100%), we compute optimal debt-equity swaps in different contexts. Simulation shows that these tools significantly increase the probability of being reimbursed of the remaining due payment in the next future.

The rest of the paper is organized as follows. In section, we analyze debt-equity swaps and their mechanisms. In section 2 we develop a parsimonious framework and highlight ad hoc designs of debt-equity swaps. Section 3 then describes the set of admissible debt equity swaps. Section 4 discusses ways to choose the optimal solution. Section 5 finally characterizes optimal solutions in various contexts.

1. The debt equity swaps mechanism

Let’s consider debt holders that face a default, say at time $T_1$, and that question how it is worth swapping (part of) their debt for equity to avoid liquidation and associated costs. Such a decision can be supported because they believe in or they know something about the economic viability of the firm or about the existence of significant growth options. This then leads to the choice of a) the amount $A$ of face value forgiven for equity, b) the proportion $\theta$ of equity they receive in exchange of $A$ and c) a set $\Theta$ of parameters that describes the rescheduled reimbursement program. For parsimony, we assume that $\Theta$ contains $\tau$ the length of the granted investment period and $F_1 - A$ the due face value to be repaid at $\tau$. In view of this, debt equity swaps are fully characterized by three structural parameters: $A$, $\theta$ and $\tau$ only because the due

---

5 It is a common practice to use the term “forgiveness” even if the swap is “fair”.

4
face value $F_1$ is known at time $T_1$. Note that, hereafter, we will interchangeably use $\tau$ or $T_2 = T_1 + \tau$, the investment horizon. It can be noticed that, by definition, $\theta$ is necessarily bounded by 100% (so $\theta \in [0,1]$). However, we will further assume that debt holders’ objective is neither to bankrupt nor to completely evict current equity holders. A reason for this may be that current equity holders/managers have special skills and unique resources concerning the business so that it could be costly to make them leave the firm. Hence, $\theta$ could be chosen arbitrarily to some specific levels comprised between 0 and 100% such that 34% and 49%. These are meaningful thresholds because in the former case debt holders have sufficient voting rights to limit the current equity holders’ decision while in the latter case, they just tolerate the leadership of current equity holders about the firm’s management decision.

Because creditors can bankrupt the firm, they should design the debt-equity swap (i.e. choose $A$, $\theta$ and $\tau$ appropriately) so as to maximize their wealth. In such a scenario, current equity holders are better off because of the rescheduling of the existing defaulting debt. They receive implicitly a claim with a strictly positive financial value, keep their power and go on running the firm. In other words, we consider here a cooperative context where stakeholders try to alleviate both bankruptcy costs and agency costs.

Clearly, this financial restructuring solution appears more and more (likely and) worthwhile, as bankruptcy costs increase. To model this, we introduce further notations. $\beta \in [0,1]$ will denote a realization rate and $V$ the firm value so that, if debt holders decide to liquidate the firm at time $T_1$, they receive only $\beta V_{T_1}$. Bris et al. (2006) have recently documented significant changes in the firm’s assets value in case of liquidation. Bris et al. (2006) write page 1264: “Chapter 7 assets drop by at least 20% in mean and 62% in median. Assuming our overly pessimistic reported-only creditor recovery, the median chapter 7 dissipates substantially all its assets even before fees are paid.”
As a result, entering a debt-equity swap implies swapping a known payoff of $\beta V_{T_1}$ for a new portfolio made of debt and part of the equity. The corresponding net gain for creditors, denoted $H$, may be written:

$$H(V_{T_1}, F_1, T_2; A, \theta) = D(V_{T_1}, F_1 - A, T_2 - T_1) + \theta C(V_{T_1}, F_1 - A, T_2 - T_1) - \beta V_{T_1}$$  \hspace{1cm} (1)$$

where $D(V_{T_1}, F_1 - A, T_2 - T_1)$ stands for the remaining debt with lower face value and $\theta C(V_{T_1}, F_1 - A, T_2 - T_1)$ the portion of new equity received in exchange of $A$. The debt equity swap is designed by setting $A$, $\theta$ and $\tau$ to proper values so as to maximize the above net gain function. To obtain a clear understanding of debt-equity swaps, additional assumptions are needed on the dynamics of the underlying firm’s assets value.

2. The framework

Our analysis adopts the continuous time framework of Black, Scholes and Merton (1973, 1974). Financial markets are perfect, complete and trading takes place continuously. There exists a riskless asset paying a known and constant interest rate denoted by $r$. There are neither taxes, neither transaction costs nor, for the moment, liquidation costs. The considered firm is financed by equity and a single debt. Without loss of generality, debt maturity is $T_1$ and face value $F_1$. The firm’s asset's value is assumed to be correctly described, under the risk neutral measure, by:

$$dV = rV dt + \sigma V dW$$  \hspace{1cm} (2)$$

where $W$ is here a Brownian motion. $\sigma$ denotes the firm’s assets volatility and stands for the level of business risk. If everything is all right at $T_1$, the payoff at time $T_1$ for equity holders is notoriously that of a call option $(\max(V_{T_1} - F_1, 0))$, for their part, in absence of liquidation costs, the debt holders receive either the promised face value if $V_{T_1} \geq F_1$ otherwise the value of the firm’s assets (or for short $\min(V_{T_1}, F_1)$). If instead there are bankruptcy costs, the value they can get is strictly lower than the assets’ value.
In this framework:

\[
D(v_t; F_i - A, T_2 - T_1) = e^{-\tau(T_z - \tau)}E_t^Q[\theta V_{T_1}^T v_{T_1} < F_i - A + (F_i - A)I_{T_1 < T_2}]
\]

(3)

and

\[
\theta C(v_t; F_i - A, T_2 - T_1) = \theta e^{-\tau(T_z - \tau)}E_t^Q[(v_{T_2} - (F_i - A))^+] \]

(4)

Standard computations under the risk neutral measure \( Q \) yields to:

\[
H(v_t; F_1, T_2; A, \theta) = -\theta V_{T_1}^T N\left[\frac{d_1^{(T_1, T_2, A, \theta)}(T_2 - T_1)}{\sigma_v}\right] + (F_i - A)N\left[\frac{d_2^{(T_1, T_2, A, \theta)}(T_2 - T_1)}{\sigma_v}\right] - \frac{\theta^{(T_1, T_2, A, \theta)}}{\sigma_v}\frac{d_2^{(T_1, T_2, A, \theta)}(T_2 - T_1)}{\sigma_v}
\]

(5)

where \( d_1(x) = \frac{\ln x + \left( r + \frac{\sigma^2}{2}\right)}{\sigma \sqrt{t}} \), \( d_2(x) = d_1(x) - \sigma \sqrt{t} \) and \( N \) is the standard cumulative distribution function. One may rewrite this equation in a more compact form:

\[
H(v_t; F_1, T_2; A, \theta) = (\theta - \beta)N\left[\frac{d_1^{(T_1, T_2, A, \theta)}(T_2 - T_1)}{\sigma_v}\right] + (1 - \theta)(F_i - A)N\left[\frac{d_2^{(T_1, T_2, A, \theta)}(T_2 - T_1)}{\sigma_v}\right]
\]

For most arbitrary values of \( A \) and \( \theta \), a detailed examination of \( H \) concludes that this is a concave function of \( T_2 \) and that its maximum value is positive. In these situations, debt holders have an incentive to choose the investment period solving:

\[
\tau(A, \theta) = \max_{T_2 \in [T_1, +\infty)} H(v_t; F_1, T_2; A, \theta) - T_1
\]

(6)

By doing so, debt holders design ad hoc debt-equity swaps. Unfortunately, things are not so simple. Clearly, the percentage \( \theta \) and the amount \( A \) are linked together; they are not independent one another at all. We now show that one of these should rather imply the other.

3. The set of admissible debt equity swaps

Clearly, debt holders cannot accept to give up part of their debt without an eye on the portion of the equity they receive and on the value this represents. This means that there exists a relation between the amount \( A \), the portion \( \theta \) and the investment period involved in the equity price. It is important to emphasize that, hereafter, we assume that creditors do not want to bankrupt the
firm nor they want to evict completely current equity holders. This assumption is clearly justified by the essence of out-of-court financial restructuring procedures. It is particularly reasonable when the current managers are necessary for the firm to survive.

Definition A debt-equity swap is *admissible* for creditors if it ensures that a/ the received portion of equity covers the forgiven amount of face value\(^6\) and b/ the resulting net gain \(H\) is maximum.

Proposition: An equity debt swap is admissible if and only if:

\[
A = \theta C(V_{T_f}, F_i - A, T(V_{T_f}, A, \theta) - T_i). \quad (7)
\]

where the optimal extension period \(T(V_{T_f}, A, \theta)\) solves:

\[
\frac{\partial H(V_{T_f}, F_i, T; A_\theta)}{\partial T} \bigg|_{T=V_{T_f}=A_\theta} = 0 \quad (8)
\]

The equation (7) ensures that, if creditors sell their part of equity immediately, they receive an amount equal to \(A\) - the amount of face value they forgive. So, the only effective effort creditors consent is to delay the distressed debt maturity. When these equations hold, \(A\) stands for both the effective amount

---

\(^6\) This condition may be viewed in fact as an *equilibrium* condition. From the point of view of debt holders, the portion \(\theta\) of equity received for a given amount \(A\) is a minimum value whereas it is a maximum value for equity holders. Alternatively, for equity holders, the amount \(A\) is a minimum value for a given portion \(\theta\) while this is a maximum for the debt holders. Readers may also note the duality of \(\theta\) and \(A\). This duality leads to an alternative strategy to design the debt equity swap optimally. We can choose either \(\theta\) or \(A\) as the “governing” parameter. The choice yields however to the same results so we choose arbitrarily the latter parameter. Finally, it must be pointed out that, if \(\theta\) and \(A\) are set to zero, the considered restructuring is a swap of debt for a new debt and the creditors’ challenge is to find the appropriate maturity extension. This latter problem has been solved by Longstaff (1990).
exchanged against equity and the value received in the form of equity. We can add that the new debt price is described by

\[ H(V_{T_i}, F_1, T; V_{T_i}, A, \theta), A, \theta) - A + \beta V_{T_i} \]

Existence of admissible debt equity swaps is ensured to the extent there exist some structural parameters \{A, \tau, \theta\} that solve the equations (7) and (8) simultaneously. To investigate whether such parameters exist, we proceed as follows. We consider first two functions of the structural parameters:

\[ \theta C(V_{T_i}, F_1 - A, \tau) - A \]

and

\[ \frac{\partial H(V_{T_i}, F_1, T; A, \theta)}{\partial T} \]

where \( T = T_i + \tau \). Then we produce, in Figure 1, three-dimensional plots of their respective zero values (under a couple of standpoints). Looking in the axis of \( \tau \), the “wall” stands for the triplets \{A, \tau, \theta\} solving \( \theta C(V_{T_i}, F_1 - A, \tau) - A = 0 \) whereas the “fall” corresponds to triplets associated to \( \frac{\partial H(V_{T_i}, F_1, T; A, \theta)}{\partial T} \bigg|_{T} = 0 \). The intersection is the set of admissible structural parameters. It indicates the structural parameters of admissible debt equity swaps. Figure 1 uses base case parameters. For visualization purposes, we scale up \( \theta \) by a factor 40 and \( A \) by a factor 2 but \( \theta \) and \( A \) range respectively from 0 to 1 and from 0 to 20.

The intersection of the two surfaces proves that there exist some triplets \{A, \tau, \theta\} that solve the equations (7) and (8) simultaneously. It reveals however that some admissible debt equity swaps require special values for the structural parameters (with, e.g. a very long investment period).
The system of two equations with three unknown variables has major consequences. It defines a structural relation between the two dual parameters $A$ and $\theta$. We can sum up consequences of proposition 1.

Consequence 1: The granted rescheduling period $\tau$ is no more a structural parameter of the debt-equity swap but a consequence of $A$ and $\theta$.

Consequence 2: $A$ and $\theta$ are structurally linked by n intricate relation described by the equations (7) and (8).

Consequence 3: In order to design a debt-equity swap, it is sufficient to fix either $\theta$ or $A$ (everything else being deduced deterministically).

To analyze admissible debt-equity swaps, we retain the following base case parameters: $F_i = 40$ for the due face value, $V_{Ti} = 20$ for the firm’s assets value, $\beta = 70\%$ and $r = 6\%$ for the actual level of the interest rate. The quasi leverage ratio is then equal to $F_i/V_{Ti} = 40/20 = 2$ which means that the default is actually severe… Figure 2 gathers four graphs where $A$ ranges from 0 to 20 whereas either the portion of new equity $\theta$ is constant (the thin lines correspond to set to 10\%, 34\%, 50\%, 67\% and 100\%) or it is implied by $A$ that is $\theta(A)$ as explained by the above consequence 3 (the bold line).

The upper left graph of Figure 1 plots the ad hoc net gain $H(T(A,\theta), A, \theta)$. The upper right one displays the “optimal” investment period implied by the other structural parameters $\tau(A,\theta)$. The lower left graph considers the probability of being fully repaid of the remaining face value computed by $PFR(A,\theta) = Q\{V_{T_i(A,\theta)} \geq F_i - A\} = N\{d_2F_i/V_{T_i(A,\theta)}(T(A,\theta) - T_i)\}$. The parameter $\theta$ impacts on $PFR(A,\theta)$ via the optimal investment period only. The lower right graph plots “flat” $\theta$s to insist that these portions are constant and unrelated to $A$.

Insert Figure 1.
These graphs mainly show that, for every level of $\theta$, debt holders are better off if they exchange a large amount of the face value. For short, as the amount $A$ gets larger, the net gains are higher, the investment period briefer and the probability of being repaid is higher. In view of these graphs, debt holders should rationally exchange the maximum value possible for equity.

The Figure 2 adds bold lines to Figure 2. These lines stand for admissible debt-equity swaps. They take into account that the portion of the equity $\theta(A)$ is a function of $A$. The net gain, the investment horizon and the probability of being reimbursed of admissible debt equity swaps are computed by $H(V_{T_i}, F_i, T(A, \theta(A)), A, \theta(A))$, $\tau(A, \theta(A))$ and $PFR(T(A, \theta(A)), A, \theta(A))$ respectively. The bottom right graph plots $\theta(A)$ implied by solving simultaneously the two equations: this is no more an arbitrary parameter.

Insert Figure 3.

The left graphs of figure 3 mainly show how valuable it is for debt holders to fix adequately the portion of the new equity they receive. The graphs on the right present the other hand. On the left side, we observe that the net gain and the probability to be fully reimbursed are significantly increased. The right side

---

7 Equity holders take also benefits from the debt equity swap because they receive a new claim whose value is positive and equal to $(1-\theta)C(V_{T_i}, F_i - A, \tau(\theta, A))$. Our framework is similar to that of Black, Scholes and Merton so this claim resembles to a call option. A key difference however rests on the investment period that is solution to equation (6).

8 Less importantly, it can be noted that, as debt holders capture a larger portion of the new equity (i.e. as $\theta$ grows to one), the optimal investment period increases to significant length and the net gain appears less sensible to the forgiven face value.

9 By virtue of footnote 10, the converse could have been investigated but we have used in figure 1 the amount $A$ as abscissa.
shows that debt holders have to wait reimbursement for a long time and must take a substantial participation in the equity. The bottom right graph highlights that the portion of equity for admissible debt-equity swaps is a non-linear function of $A$. It reaches rapidly the successive levels of participation we choose (10%, 34%, 50% and 67%). Figure 3 illustrates overall how different is our optimal method to design admissible debt equity swaps compared to the ad hoc specification of Figure 1. Note that we do not analyze further the Figure 3 because we expose fifty cases in the last section.

At the end of this analysis, a natural question arises on how to choose the optimal debt-equity swap. This is the point we discuss now.

4. In quest of the optimal set of debt equity swap

Because we assume that debt holders do not want to take over the firm, they must find the optimal debt equity swap among the set of admissible candidates. Because this set has been derived by solving a system of two equations with three unknown variables, an additional constraint should certainly be imposed. Figure 3 indicates that there is no specific and endogenous $A$ to choose. This means that a supplementary condition must be considered to select the optimal debt equity swap. We propose in this section various possible specifications.

First of all, debt holders can choose an arbitrary amount to exchange for equity and make their decision on $A^*$. The structural parameters of the optimal debt equity swap are then obtained along the lines described in the precedent section. For instance, the associated net gain function is computed by:

$$H^* = H(V^*, F, T(A^*, \theta(A^*)), \lambda, \theta(A^*))$$ where $T(A^*, \theta(A^*)) = T^* + \tau(A^*, \theta(A^*))$. 

Other ways to proceed consist in deducing $A^*$ from a decision on the participation in the new equity or the length for the investment period.

Debt holders can retain a maximum investment period $\tau^*$ voluntarily or for legal and institutional purposes\(^\text{10}\). The amount $A^*$ is then simply deduced from this by using, for instance, the upper right graph of figure 3. Another equivalent way to proceed consist in using the figure 2. With Figure 3, one finds $A^* = \tau^{-1}(\tau^*)$ and then we can compute $\theta(A^*) = \theta(\tau^{-1}(\tau^*))$ and the net gain of the optimal debt equity swap: $H^* = H(V_{t_1}, F_i, T^*; \tau^{-1}(\tau^*), \theta(\tau^{-1}(\tau^*)))$ where $T^* = T_i + \tau^*$.

A third approach consists in considering a target participation $\theta^*$. Theoretically, $\theta^*$ can be set to any values in the interval $[0,1]$. Some values, however, are particularly more meaningful than others. 10%, 34%, 50% and 67% are the values we retain in the graphs. Armed with such a target value $\theta^*$, debt holders can infer the value from the figure 3 and for the amount $A$ from the lower right graph. They find $A^*$ such that $\theta^* = \theta(A^*)$ i.e. $A^* = \theta^{-1}(\theta^*)$ because $\theta(A)$ is a one-to-one function of $A$\(^\text{11}\). The resulting debt holders net gain is then: $H^* = H(V_{t_1}, F_i, T(\theta^{-1}(\theta^*), \theta^*); \theta^{-1}(\theta^*) \theta^*)$ where $T(\theta^{-1}(\theta^*) \theta^*) = T_i + \tau(\theta^{-1}(\theta^*) \theta^*)$.

\(^{10}\) Gilson et al. (1990) recall that “Banks are constrained from holding significant blocks of stocks […] exceptions are granted when stock is obtained in a debt restructuring. […] In general, banks must divest their stockholdings after approximately two years although extensions are possible.” In view of James’s (1990) results, extensions are very frequent because “banks continue to hold substantial amount of common stock two years after [the restructuring]”. Only 2 banks for 27 divest…

\(^{11}\) The footnote 10 points out that, due to the duality between $A$ and $\theta$, there exists a couple of alternative ways to design the set of admissible debt equity swaps. Clearly, the “best way” to proceed depends on the final criterion debt holders retain for choosing their optimal solution. Here we choose $A$, so the simplest way to design swap is when debt holders make their decision on an arbitrary amount $A^*$ (compared to what occurs if they consider a $\theta$ – criterion).
It is worth noting that the optimal debt-equity swap provided by the above approach is far from being equivalent to an *ad hoc* debt-equity swap that uses the target participation. The *ad hoc* debt-equity swap solves

$$\max_{A \in [0,F_i-V_f]} H(V_{\tau_i}, F_i, T(A); A, \theta^*)$$

where $T(A) = T_i + \tau(A)$ and $\theta^*$ is a constant participation. The upper left graph of the figure 1 shows that, for a constant $\theta$ ($\theta = \theta^*$), the net gain function $H(V_{\tau_i}, F_i, T(A); A, \theta^*)$ is increasing with respect to the amount $A$. Debt holders should rationally choose to forgive the maximum value $A_{\text{max}}^{\text{debt}} = F_i - V_f$. By contrast, our previous analysis on admissible debt-equity swaps defends that this solution is not optimal because debt holders are not fairly compensated for the exchange. The complete picture is given by the lower right graph of the figure 3 that provides the optimal amount to consider. The next section undertakes a in-depth analysis of the optimal debt equity swaps obtained by such a $\theta$– criterion.

5. **Analysis of optimal debt equity swaps**

Let’s assume that debt holders favour the $\theta$– criterion to design the optimal debt-equity swaps. To analyze the design of debt-equity swaps, we consider fifty different default contexts including that of the precedent figures. In every default situation, we design optimal debt equity swaps by following the lines exposed in the previous section. We denote by $\theta^*$ the target participation and we characterize the optimal debt equity swaps by $A^* = \theta^{-1}(\theta^*)$, $\tau^*$, $H^* = H(V_{\tau_i}, F_i, T(\theta^{-1}(\theta^*), \theta^*), T(\theta^{-1}(\theta^*), \theta^*) = T_i + \tau^*$ and $PFR^* = PFR(T(\theta^{-1}(\theta^*), \theta^*), T(\theta^{-1}(\theta^*), \theta^*)$ which correspond respectively to the amount

The reverse conclusion would arise if we have followed the alternative way. Of course, both ways leads to the same results. We present one approach only but note that most of our results (in particular Table 1) have been computed and checked by the two approaches.
of face value exchanged for equity, the investment period, the net gain and the probability that the remaining face value will be fully repaid in the next future.

Table 1 describes the optimal debt-equity swaps, debt holders should consider if they target a 50%– participation in the new equity of the firm. This table provides other structural parameters of the optimal debt equity swap (i.e. $A^*$, $\tau^*$), as well as $H^*$ and $PFR^*$. As discussed earlier, the length of the investment period $\tau^*$ can be interpreted here as the effort consented by debt holders. By virtue of equation (7), $A^*$ is the value of new equity received by debt holders. Interestingly, because the new equity is halved and shared among parties ($\theta^*=50\%$), $A^*$ is also the value received by equity holders.

Insert Table 1.

Table 1 exposes optimal debt equity swaps in fifty different contexts. Let’s first of all examine a specific situation where, say, $\beta=80\%$ and $V_\zeta=26$. Simulations show that the net gain is $H^*=0.907$ so debt holders are better off to swap debt for equity. In addition, the new financial set up appears rather similar to the distressed one. The face value of the new debt is only slightly lower than before. It is equal to: $F^*=F_i-A^*=40–1.10=38.9$. The effort of debt holders may be considered as moderate since creditors just have to grant a delay of $\tau^*=3.65$. More interestingly, the probability of being fully repaid of the remaining face value $F^*$ is $PFR^*=25.08\%=\frac{1}{4}$. This value may appear significant if we remind that debt holders have just faced a default and that they can incur a loss (with certainty) if they decide liquidation immediately. Beyond this specific context, the Table shows that, whatever the default severity and liquidation costs, it is valuable for creditors to take over the firm. In every case, necessary concessions $\tau^*$ appear rather weak (once again compared to an immediate
liquidation of the firm’s assets) proving that debt-equity swap is a very powerful solution in the hand of creditors. Simulations moreover indicate that $H^*$ increases as the default gets softer and grows as the liquidation costs increases. $PR^*$ ranges from more than 5% to nearly 60%. Higher for softer default and higher liquidation costs, it appears that the realization rate has a significant effect. A reason for this is that $\beta$ impacts critically on the optimal investment period. It is also interesting to inspect in some details $A^*$, the amount forgiven by creditors. A reason for this is that this is the value the equity holders receive in the restructuring. The debt equity swap leaves indeed half of the equity to equity holders, for a total value of $A^*$. $A^*$ is the debtors’ net gain as well because the equity is worth nothing in default. The key result of table 1 is that $A^*$ is not a simple function of the liquidation costs and severity of default. The amount $A^*$ is a humped function of the severity of default. For illustration, when $\beta = 60\%$, the largest $A^*$ (2.94) corresponds to $V_T = 30$. This property of $A^*$ suggests a strategic behaviour for debtors who may be better off to invest in projects with negative NPV few times before a default. Overall, we can retain that creditors are better off by swapping debt for equity but also that the same is financially true for equity holders. This solution could however hurt these latter because such an offering means they loose half of their ownership.

**Conclusion**

This paper analyses debt-equity swaps with a special focus on their design. Assuming that debt holders do not want to take over the firm, we solve the debt holders’ design problem. We exploit three parameters that characterize debt equity swaps. These are the amount of exchanged debt, the participation level in the new equity and the duration of the investment period. We find, in our parsimonious contingent claim analysis, that there exists a set of admissible solutions. After discussing different ways to choose debt equity swaps, we
derive the optimal restructuring given that creditors target a specific participation in the new equity. Simulation shows that the debt equity swap is an efficient method for creditors to restructure the distressed debt. Clearly, our parsimonious framework neglects some issues related to debt-equity swaps and, in particular, the struggle of equity holders not to lose their ownership, but the sort of computation we run constitutes a quantitative benchmark for shareholders to approach the final negotiation.

References


These graphs show, under a couple of viewpoints, the triplets \( \{A, \tau, \theta\} \) solving
\[
\theta C(V_{T_i}, F_i - A, T - T_i) - A = 0
\]
(the “wall” in the \( \tau \)-axis) and those solving
\[
\frac{\partial H(V_{T_i}, F_i, T, A, \theta)}{\partial T} \bigg|_{T^*} = 0
\]
where \( T = T_i + \tau \) (the “fall” in the \( \tau \)-axis). The intersection corresponds to the structural parameters of admissible debt equity swaps. Other parameters are: \( V_{T_i} = 20 \), \( F_i = 40 \), \( \sigma = 20\% \), \( r = 6\% \), \( \beta = 70\% \). Please remark that, for visualization purposes, we multiply \( \theta \) by 40 and \( A \) by 2 so \( \theta \) ranges from 0 to 1 and \( A \) from 0 to 20.
Figure 3: Designing debt-equity swaps: the net gain, the investment period and the probability of being fully repaid as functions of the forgiven face value when the received portion of the new equity is arbitrarily or optimally chosen.

These graphs are identical to those of Figure 1 except that we insert bold lines to draw \( H(T(A), A), \tau(A), PFR(A) \) and \( \theta(A) \) respectively. Other parameters are: \( V_0 = 20, F_0 = 40, \sigma = 20\%, \ r = 6\%, \ \beta = 70\% \).
Figure 3: Designing debt-equity swaps: the net gain, the investment period and the probability of being fully repaid as functions of the forgiven face value when the received portion of the new equity is arbitrarily or optimally chosen.

These graphs are identical to those of Figure 1 except that we insert bold lines to draw $H(T(A), A)$, $\tau(A)$, $PFR(A)$ and $\theta(A)$ respectively. Other parameters are: $V_t = 20$, $F_t = 40$, $\sigma = 20\%$, $r = 6\%$, $\beta = 70\%$. 
Table 1: Optimal debt equity swaps ensuring a target participation of $\theta^* = 50\%$

<table>
<thead>
<tr>
<th>Realization Rate $\beta$</th>
<th>20</th>
<th>22</th>
<th>24</th>
<th>26</th>
<th>28</th>
<th>30</th>
<th>32</th>
<th>34</th>
<th>36</th>
<th>38</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^*$</td>
<td>0.15</td>
<td>0.19</td>
<td>0.24</td>
<td>0.29</td>
<td>0.35</td>
<td>0.40</td>
<td>0.46</td>
<td>0.51</td>
<td>0.54</td>
<td>0.50</td>
</tr>
<tr>
<td>$\tau^*$</td>
<td>3.08</td>
<td>2.67</td>
<td>2.30</td>
<td>1.95</td>
<td>1.62</td>
<td>1.31</td>
<td>1.02</td>
<td>0.75</td>
<td>0.48</td>
<td>0.22</td>
</tr>
<tr>
<td>$H^*$</td>
<td>0.056</td>
<td>0.079</td>
<td>0.109</td>
<td>0.150</td>
<td>0.203</td>
<td>0.274</td>
<td>0.372</td>
<td>0.510</td>
<td>0.715</td>
<td>1.061</td>
</tr>
<tr>
<td>$PFR^*$</td>
<td>0.0535</td>
<td>0.0686</td>
<td>0.0865</td>
<td>0.1078</td>
<td>0.1331</td>
<td>0.1633</td>
<td>0.1988</td>
<td>0.2440</td>
<td>0.3000</td>
<td>0.3751</td>
</tr>
<tr>
<td>90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A^*$</td>
<td>0.81</td>
<td>0.91</td>
<td>1.01</td>
<td>1.10</td>
<td>1.18</td>
<td>1.25</td>
<td>1.29</td>
<td>1.29</td>
<td>1.21</td>
<td>0.96</td>
</tr>
<tr>
<td>$\tau^*$</td>
<td>5.72</td>
<td>4.98</td>
<td>4.29</td>
<td>3.65</td>
<td>3.05</td>
<td>2.48</td>
<td>1.94</td>
<td>1.42</td>
<td>0.91</td>
<td>0.41</td>
</tr>
<tr>
<td>$H^*$</td>
<td>0.498</td>
<td>0.614</td>
<td>0.749</td>
<td>0.907</td>
<td>1.094</td>
<td>1.318</td>
<td>1.588</td>
<td>1.926</td>
<td>2.365</td>
<td>2.987</td>
</tr>
<tr>
<td>$PFR^*$</td>
<td>0.1765</td>
<td>0.1997</td>
<td>0.2244</td>
<td>0.2508</td>
<td>0.2791</td>
<td>0.3097</td>
<td>0.3430</td>
<td>0.3798</td>
<td>0.4210</td>
<td>0.4674</td>
</tr>
<tr>
<td>80</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A^*$</td>
<td>1.62</td>
<td>1.76</td>
<td>1.89</td>
<td>1.99</td>
<td>2.07</td>
<td>2.12</td>
<td>2.12</td>
<td>2.04</td>
<td>1.84</td>
<td>1.36</td>
</tr>
<tr>
<td>$\tau^*$</td>
<td>8.04</td>
<td>7.02</td>
<td>6.07</td>
<td>5.19</td>
<td>4.35</td>
<td>3.56</td>
<td>2.80</td>
<td>2.06</td>
<td>1.34</td>
<td>0.61</td>
</tr>
<tr>
<td>$H^*$</td>
<td>1.343</td>
<td>1.584</td>
<td>1.851</td>
<td>2.149</td>
<td>2.486</td>
<td>2.868</td>
<td>3.309</td>
<td>3.828</td>
<td>4.459</td>
<td>5.262</td>
</tr>
<tr>
<td>$PFR^*$</td>
<td>0.2802</td>
<td>0.3038</td>
<td>0.3280</td>
<td>0.3527</td>
<td>0.3783</td>
<td>0.4049</td>
<td>0.4326</td>
<td>0.4616</td>
<td>0.4916</td>
<td>0.5197</td>
</tr>
<tr>
<td>70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A^*$</td>
<td>2.41</td>
<td>2.58</td>
<td>2.73</td>
<td>2.84</td>
<td>2.92</td>
<td>2.94</td>
<td>2.90</td>
<td>2.76</td>
<td>2.44</td>
<td>1.76</td>
</tr>
<tr>
<td>$\tau^*$</td>
<td>10.14</td>
<td>8.88</td>
<td>7.70</td>
<td>6.60</td>
<td>5.56</td>
<td>4.57</td>
<td>3.61</td>
<td>2.68</td>
<td>1.75</td>
<td>0.80</td>
</tr>
<tr>
<td>$PFR^*$</td>
<td>0.3618</td>
<td>0.3838</td>
<td>0.4059</td>
<td>0.4282</td>
<td>0.4506</td>
<td>0.4734</td>
<td>0.4964</td>
<td>0.5195</td>
<td>0.5416</td>
<td>0.5571</td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A^*$</td>
<td>3.14</td>
<td>3.33</td>
<td>3.50</td>
<td>3.62</td>
<td>3.69</td>
<td>3.70</td>
<td>3.63</td>
<td>3.42</td>
<td>3.01</td>
<td>2.14</td>
</tr>
<tr>
<td>$\tau^*$</td>
<td>12.07</td>
<td>10.59</td>
<td>9.21</td>
<td>7.92</td>
<td>6.70</td>
<td>5.52</td>
<td>4.39</td>
<td>3.28</td>
<td>2.17</td>
<td>1.01</td>
</tr>
<tr>
<td>$PFR^*$</td>
<td>0.4265</td>
<td>0.4467</td>
<td>0.4667</td>
<td>0.4865</td>
<td>0.5062</td>
<td>0.5258</td>
<td>0.5452</td>
<td>0.5639</td>
<td>0.5803</td>
<td>0.5868</td>
</tr>
</tbody>
</table>

This table presents the amount of face value optimally ‘forgiven’ by the debt holders (on the first row), the associated optimal extension period (on the second row), their net gains from optimally swapping debt for equity (on the third row) and the probability of being fully repaid (on the fourth row), for different values of the realization rate ($\beta$ in percentage) and values of the firm assets at the time of default $V_t$. Other parameters are: $F_t = 40$, $\sigma_1 = 20\%$, $r = 6\%$. 