Crash risk in the cross section of stock returns

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Abstract

Recent evidence indicates that the risk of a systematic market-wide crash is priced. In this paper we examine the importance of this crash risk premium for the cross section of stock returns. We show that the CAPM should be augmented with a separate crash risk factor, if the sensitivity of stocks to market-wide crashes is different from their sensitivity to normal market movements. For the CRSP cross section of US stocks, this crash risk factor is important, as we find that a portfolio of stocks with a high market crash sensitivity pays a significant positive expected return of 2.3\% to 4.0\% per annum, after correcting for normal market risk. This extra return cannot be explained by other risk factors, including coskewness and cokurtosis. Finally, the addition of crash risk portfolios improves the explanation of the cross section of stock returns, in particular when stocks are sorted on momentum.

Key words: asset pricing, crash risk, jump-diffusion models
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1 Introduction

Crashes have dramatic consequences for investors. Because comovements of assets become stronger when crashes occur (as shown by Longin and Solnik (2001), Ang and Chen (2002) and Kole et al. (2006)), these consequences are hard to diversify and difficult to evade. For these reasons, financial theory predicts that investors receive a premium for the crash risk that they bear. If asset returns exhibit different characteristics during a crash than during quiet periods, or investors are particularly averse to crashes, the crash risk premium should be distinguishable from the risk premium for quiet times. Knowledge about this premium and its structure can then increase our understanding of investor preferences and the expected returns of assets both over time and in the cross section.

In this paper, we examine whether we can identify a separate risk premium for crash risk in the cross section of stock returns. Up to now, evidence of a premium for crash risk is largely based on options on stock market indexes (see Andersen et al., 2002; Bates, 1991, 2000). However, Bakshi et al. (2003) show that the distributions of stock returns implied by individual stock options differ from the implied distribution for market index returns, as they are less skewed. Chen et al. (2001) report wide variation in the skewness of stock return distributions. This evidence can indicate that individual stocks vary in their sensitivity to market crashes, and differ from the market in aggregate. As a consequence, the expected returns of stocks can vary with their exposure to market crash risk.

To investigate systematic crash risk in asset prices, we derive an extension of the Capital Asset Pricing Model (CAPM) that includes a premium for crash risk. The distinctive characteristic of crashes in relation to normal movements is their sudden and speedy occurrence. Therefore, we augment the standard process for stock prices, containing a drift term and Brownian innovations with a downward jump component, based on Bates (2001). The occurrence of jumps follows a Poisson process, which means that during each interval the stock price can suddenly fall by a large amount. To capture investors’ aversion to such large losses we include a specific crash aversion discount in the utility function. Individual stocks can vary in their tendency to crash given that the market crashes. The expected return for a stock contains a reward for crash risk proportional to its sensitivity for it. We derive that a stock’s sensitivity to crash risk consists of two parts: the probability that it
crashes, given that the market crashes, and a ratio of the crash magnitude of the asset to that of the market. The premium for crash risk is a function of general risk aversion and crash aversion.

We use this model to set up an empirical analysis of crash risk in the cross section of stock returns. Based on copulas, we derive three measures for the conditional crash likelihood that shows up in an asset’s sensitivity to market crashes. The intuition behind these three measures is that crashes can explain the difference between the actual dependence of an asset and the market, and the correlation-implied dependence (see Hartmann et al., 2004; Longin and Solnik, 2001). If we find that an asset exhibits stronger dependence with the market for extreme downward movements than its correlation with the market can explain, we interpret that difference as an exposure to crash risk. For each stock we construct estimates for the values of the three measures and use them to sort the stocks into value-weighted portfolios.

Based on the CRSP-database from June 1964 to November 2003, we provide evidence that crash risk shows up as a separate premium in the cross section of stock returns. After correcting for market risk as in the traditional CAPM, portfolios with stocks that score in the top 33% of exposure to crash risk yield on average an extra significant return. This return ranges from 2.3% to 4% per annum, depending on which measure the stocks are sorted. These extra returns cannot be explained by established factors, such as size, value or momentum. We do find that these portfolios are related to the coskewness effect of Harvey and Siddique (2000), and the cokurtosis effect of Dittmar (2002), but the extra returns remain significant after correcting for these effects. The portfolios with stocks that exhibit little or no exposure to crash risk do not pay an extra significant average return.

We find evidence that a crash risk factor contributes to the explanation of the cross-section of stock returns. For portfolios based on momentum, which constitute the largest anomaly over our sample period, this added value is most clear. The traditional CAPM produces pricing errors that are jointly significant, but this significance vanishes after including a crash risk factor. For portfolios sorted on industry, size or value versus growth we find small improvements, but the traditional CAPM suffices to explain these cross sections of portfolio returns. An analysis of the entire cross section of stock returns also indicates that pricing errors and their standard errors are lower after addition of a crash
This paper adds to the ongoing debate on asset pricing. We investigate the presence of a factor that can be related directly to risk, which distinguishes our research from the more data based approaches underlying the size and value factors of Fama and French (1993, 1995) and the momentum factor of Jegadeesh and Titman (1993) and Carhart (1997). Our research complements the literature that extends the traditional CAPM with higher order moments, such as coskewness in Harvey and Siddique (2000) and Barone Adesi et al. (2004), or cokurtosis in Dittmar (2002) and Christie-David and Chaudhry (2001). Coskewness and cokurtosis can be interpreted as proxies for the tendency of assets to crash conditional on a market crash. Indeed, we find a relation between the crash risk portfolios on the one hand, and the coskewness and cokurtosis portfolios on the other hand. However, for the period we consider, the returns on the coskewness and cokurtosis hedge portfolios are insignificant. This can indicate that coskewness and cokurtosis are imperfect proxies for crash risk.

Our study also contributes to the wide literature on non-linear dependence. By now, it is well known that correlation, and hence linear dependence, fall short in accurately describing actual dependence between assets. Copula have proved a successful alternative to the traditional correlation approach. While the implications and importance of copulas for risk management and asset allocation have been documented before (Kole et al., 2006; Poon et al., 2004; Glasserman et al., 2002; Embrechts et al., 2002, see e.g.), this paper is the first to apply copulas in asset pricing. Our findings clearly demonstrate that the assumption of linear dependence can lead to a serious underestimate of the risk premium on an asset.

This article is structured as follows. In Section 2 we derive the CAPM extended with crash risk. In Section 3 we present the measures for conditional crash likelihood. Section 4 discusses the portfolio formation based on these measure and reports the relation of the portfolios with other risk factors. Section 5 considers the cross section of stock returns in relation to crash risk, while section 6 concludes.
2 Extending the CAPM with crash risk

The traditional Capital Asset Pricing Model (CAPM) as put forward by Sharpe (1964) and Lintner (1965) posits that the following equilibrium relation between the excess return $R_{e,i,t+1}$ on asset $i$ and the excess return on the wealth portfolio $R_{e,w,t+1}$ holds at time $t$:

$$E_t[R_{e,i,t+1}] = \beta_{i,t} E_t[R_{e,w,t+1}],$$

(1)

where $E_t[.]$ denotes the conditional expectation given information available at time $t$. In this expression, $\beta_{i,t}$ measures the sensitivity of the asset’s excess return with respect to the return on the wealth portfolio. In empirical work, the wealth portfolio is commonly approximated by the market portfolio.

The CAPM can be derived straightforwardly by assuming an endowment economy in which the representative agent has power utility and the systematic uncertainty stems from diffusion processes. While such an approach provides a nice starting point for studying asset pricing, it is too restrictive to study the influence of crash risk on asset prices. Following Bates (2001), we propose modifications of the standard approach to address important drawbacks. We use the insights of the extended CAPM to steer the empirical research in the remainder of this paper.

We derive the extended CAPM from the standard CAPM setting in which we assume an endowment economy with a finite-lived representative agent who consumes at the final date $T$. In this economy $n+1$ assets are available. The first asset is a contract that provides a certain pay-off at the end date. The other $n$ assets entitle the owner to an uncertain final pay-off, denoted by $X_{iT}, i = 1, \ldots, n$. We assume that the riskless asset is in zero net supply, while each of the risky assets have a net supply of unity. The market then consists of the sum of the uncertain pay-offs, which we call the market claim and denote as $X_{mt}$. The CAPM enables us to derive the price dynamics of these assets over time, based on the dynamics of the underlying processes. We use $S_{it}$ to denote the time $t$ price of the asset $i$, and $S_{mt}$ for the price of the market claim.

As a first extension we add jumps as a source of systematic risk to the diffusions present

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1See Cochrane (2001, Ch. 9.1) for other sets of assumptions that lead to the CAPM.
in the standard CAPM. We assume that the growth rate of the final market pay-off follows

\[ d\log X_m = \mu_m dt + \sigma_m dZ_m + \kappa_m dN_m, \]  

(2)

where \( \mu_m dt \) is the drift rate, \( Z_m \) is a Wiener process, \( N_m \) is a Poisson process with arrival rate \( \lambda_m \), \( \kappa_m \) is the jump size, which we assume to be negative, and \( \sigma_m^2 dt \) is the variance rate of the process, conditional on no jumps occurring over \( dt \). We assume that the Wiener process and the Poisson process evolve independently.

Poisson processes are essential to capture the sudden price decreases during crashes. Empirical evidence indicates that stock prices are not only driven by diffusion processes but exhibit negative jumps as well (see Das and Uppal, 2004, and references therein). Moreover, evidence from option markets in Andersen et al. (2002) and Bates (2000, 1996, 1991) indicates that investors expect downward jumps to occur and require a premium for this risk. Stock price processes that are constructed with diffusions only (including stochastic volatility) cannot generate the distributions that are implied by option prices. This implies that jumps have a systematic component, which should show up in the pricing kernel.

Second, we add a crash discount to the power utility function of the representative agent, formulated indirectly in terms of wealth:

\[ U(W_t, N_{mt}) = E_t \left[ e^{\delta N_{mt} W_t^{1-\gamma}} - 1 \right], \quad \gamma > 1, \quad \delta \geq 0, \]  

(3)

where \( W_t \) denotes the investor’s wealth at time \( t \), \( N_{mt} \) denotes the number of market crashes up to time \( t \), \( \gamma \) is the investor’s coefficient of relative risk aversion and \( \delta \) reflects his crash aversion.

Standard utility functions are ill-suited to capture investors’ aversion to downside risk.\(^2\) In our setup an extra crash multiplies the value of the utility function with a factor \( e^{\delta} > 0 \), reducing the value of the utility function (for \( \gamma > 1 \) we have \( W_t^{1-\gamma}/(1 - \gamma) < 0 \)). This setting implies loss aversion in a limited sense, since it applies only to crash losses and not to losses in general. However, this is not a substantial limitation, as we are specifically

interested in the effect of crash risk. Furthermore, in the absence of market crashes $N_m = 0$, our model design leads to the traditional CAPM, providing clear insights what the changes imply.

The third extension of the traditional CAPM posits a structure for the stochastic processes that underlie the specific pay-off $X_{iT}$ that is similar to the structure for the process for the market pay-off in Eq. (2):

$$d \log X_i = \mu_i dt + \sigma_i dZ_i + \kappa_i dN_i,$$

where $Z_i$ is a Wiener process, and $N_i$ is a Poisson process with arrival rate $\lambda_i$, independent from $Z_i$. The Wiener processes $Z_m$ and $Z_i$ are related via $E[dZ_i dZ_m] = \rho_{i,m} dt$. The Poisson process $N_i$ is different from $N_m$, but not independent from it. We assume that the joint process $N = \left( N_m \ N_i \right)'$ evolves according to:

$$dN = \begin{cases} 
(1 1)' & \text{with probability } \lambda_{i,m} dt \\
(1 0)' & \text{with probability } (\lambda_m - \lambda_{i,m}) dt \\
(0 1)' & \text{with probability } (\lambda_i - \lambda_{i,m}) dt \\
(0 0)' & \text{with probability } (1 - \lambda_i - \lambda_m + \lambda_{i,m}) dt.
\end{cases}\tag{5}$$

To guarantee that each arrival rate falls in the $[0,1]$ interval, we impose the restriction $\lambda_{im} \leq \min \{ \lambda_i, \lambda_m \}$. The processes $N_m$ and $N_i$ can be interpreted as the marginal processes of the process $N$ (with marginal arrival rates equal to $\lambda_m$ and $\lambda_i$, respectively). $N_m$ and $N_i$ are independent iff $\lambda_{i,m} = \lambda_i \lambda_m$.\(^3\)

The structure of this bivariate Poisson process captures our basic idea that individual assets need not all behave in the same way when the market encounters a crash. A value of $\lambda_{i,m}/\lambda_m$ close to one implies that an individual asset has a high likelihood to crash if the market crashes, whereas a value for $\lambda_{i,m}/\lambda_m$ close to zero implies that this probability is negligible. As we show later, this conditional probability is crucial for the crash risk premium present in an individual asset. This structure distinguishes our design from Ho et al. (1996), where one jump process is present in all processes underlying the assets, and

\(^3\)Independence is equivalent with $Pr[dN_i = 1|dN_m = 1] = \lambda_{i,m}/\lambda_m dt = \lambda_i dt = Pr[dN_i = 1]$.\(^7\)
from Merton (1971), who studies asset pricing in the presence of idiosyncratic jumps in the processes underlying the assets.

Based on these assumptions we derive the pricing kernel and the price processes of the assets in the economy. We provide the derivation in Appendix A and discuss the resulting equilibrium equations for the expected returns here. The instantaneous expected excess return on the market asset equals

$$E_t \left[ R_{mt} \right] \equiv E_t \left[ \frac{dS_{mt}}{S_{mt}} \right] = \gamma \sigma_m^2 dt + \lambda_m \left( e^{\delta - \gamma \kappa_m} - 1 \right) (1 - e^{\kappa_m}) dt. \quad (6)$$

The first term is the risk premium that the agent requires in the traditional CAPM setting. In our setting it is the risk premium associated with diffusion risk. The price of a unit of diffusion risk \( \sigma_m^2 \) is \( \gamma \). The second term reflects crash risk. Its expression is more complicated than the expression for diffusion risk, but it has a similar structure. The effect of a crash consists of its probability \( \lambda_m \) times its impact on wealth, being a decrease with a factor \( 1 - e^{\kappa_m} \) (the exponent arises because the crash takes place in the growth rate of \( X_{mT} \)). To find the premium associated with crash risk, this expression is multiplied by \( e^{\delta - \gamma \kappa_m} - 1 \), which can be interpreted as the price of one unit of crash risk. It is a function of general risk aversion \( \gamma \) and an extra term \( \delta \) capturing crash aversion.

The expected excess return on asset \( i \) as underlying equals

$$E_t \left[ R_{it} \right] \equiv E_t \left[ \frac{dS_{it}}{S_{it}} \right] = \frac{\sigma_i}{\sigma_m} \rho_{im} \zeta^d dt + \frac{1 - e^{\kappa_i}}{1 - e^{\kappa_m}} \frac{\lambda_{i,m}}{\lambda_m} \zeta^c dt, \quad (7)$$

where we use \( \zeta^d \equiv \gamma \sigma_m^2 \) and \( \zeta^c \equiv \lambda_m \left( e^{\delta - \gamma \kappa_m} - 1 \right) (1 - e^{\kappa_m}) \). The first term is again a reward for the diffusion risk that the contract on \( X_{iT} \) entails. It has the same structure as the traditional CAPM. The expression \( \sigma_i/\sigma_m \cdot \rho_{i,m} \) gives the asset’s \( \beta \) with respect to the market as in Eq. (1). Note that this \( \beta \) gives a sensitivity, conditional on no crashes occurring, which is why we call it a diffusion-\( \beta \). The second term covers the premium for crash risk. Similar as the diffusion risk premium, it consists of the risk premium for market crashes multiplied with a sensitivity factor. This sensitivity factor can be easily compared to the diffusion-\( \beta \). The ratio \( (1 - e^{\kappa_i})/(1 - e^{\kappa_m}) \) reflects the relative magnitude of a crash in \( X_{iT} \) in the same way as the ratio \( \sigma_i/\sigma_m \) give the relative magnitude of the diffusion in \( X_{iT} \). The conditional probability \( \lambda_{i,m}/\lambda_m \) reflects the dependence between crashes in \( X_{iT} \) and market crashes and can be compared to the parameter \( \rho_{i,m} \), which corresponds with
the dependence of the two diffusion processes. Consequently, we interpret the product of the magnitude ratio and the conditional probability as the asset’s crash-β.

Equations (6) and (7) offer the main insights of the crash-CAPM. Under the crash-CAPM, the expected return on a stock can be split in a part related to its sensitivity to the market during normal periods and a part related to its sensitivity to the market in times of a market crash. The premium on crash risk reflects the “normal” aversion to the risk that crashes entail and the extra aversion that agents have to crash losses. As a consequence, the crash risk premium can be quite pronounced.

3 Conditional crash likelihood

The insights of the crash-CAPM are useful in guiding the empirical research in this paper. We want to shed more light on two issues. First, we investigate whether we can identify a crash risk premium in the cross section of stock returns. Second, we examine whether crash risk helps explaining the cross-sectional variation of stock returns. To answer the first question we use the common technique of sorting stocks into portfolios, in this case based on their sensitivity to market crashes (Harvey and Siddique, 2000, follow the same approach for coskewness). To answer the second question we use those portfolios again. We do not base the answers on direct estimation of the Crash-CAPM, but use the portfolio approach instead, because it needs less assumptions.

In this section we propose three measures to determine a stock’s market crash sensitivity. According to Eq. (7) the sensitivity consists of two components: a ratio of crash magnitude and a conditional probability of a crash, given that the market crashes. Our measures concentrate on that conditional probability, and are based on the difference between the actual dependence and the dependence implied by the diffusion processes. The conditional crash probability affects the dependence between an asset and the market. If it equals zero, this dependence originates solely from the dependence in the diffusion processes, which is measured by the correlation coefficient \( \rho_{c,m} \). If the conditional probability is larger than zero, the actual dependence between an asset and the market reflects both the general

\[\text{We expect that the cross-sectional variation in stocks’ crash } \beta \text{ stems mainly from this component, in particular if we take into account that crash magnitudes are stochastic.}\]
dependence from the diffusion processes and the dependence due to crashes. Indeed, empirical evidence finds that the dependence between stocks and markets increases for more extreme observations.\textsuperscript{5} In other words, if the proportion of observations in a sample that qualify as crashes increases, the observed dependence becomes stronger.

The three measures are derived from copulas, because they constitute a flexible and powerful tool to model dependence. A copula is a function that models the dependence between random variables. It returns the joint cumulative probability of a set of events as a function of the cumulative marginal probabilities of each event.\textsuperscript{6} Since we are interested in dependence between an asset and the market, we will use bivariate copulas in this paper. This means that we model the joint distribution of the return on asset $i$ and the market return with a copula $C()$ as

$$\Pr(R_i \leq r_i, R_m \leq r_m) = C(\Pr(R_i \leq r_i), \Pr(R_m \leq r_m)).$$ \textsuperscript{(8)}

The dependence due to the diffusion processes implies a Gaussian copula, which is related to the normal distribution. If the conditional crash probability is high, the actual dependence will deviate from the diffusion-implied Gaussian copula, and this deviation will be larger for a larger conditional crash probability. We exploit this idea in the three measures that we construct. The first measure is based on the difference between the empirical copula and the Gaussian copula. The second and third measure use the Student’s $t$ copula. We discuss the construction of the measures in more detail in the next subsection. Since copulas model only the dependence between random variables, we first discuss how the marginal return distribution can be modeled.

### 3.1 Marginal models

The marginal distributions that Eqs. (A3) and (A6) imply for the market return and the individual asset return, respectively, would be normal distributions in the absence of crash risk. Its presence will lead to distributions that deviate from normal distributions and have

\textsuperscript{5}See among others Hartmann et al. (2004), Bae et al. (2003), Ang and Chen (2002) and Longin and Solnik (2001).

\textsuperscript{6}Copula theory is discussed in general terms in Joe (1997) and Nelsen (1999), and in a financial setting by Cherubini et al. (2004) and Bouyé et al. (2000).
negative skewness and fat tails. As a consequence, we model the marginal distributions
by a skewed Student’s $t$ distribution. It can be constructed from the regular Student’s $t$
distribution by the method proposed by Fernández and Steel (1998) as shown by Lambert
and Laurent (2001) and de Jong and Huisman (2000). Its density function is given by:

$$
\psi_{sk}(x; \mu, \sigma, \nu, \xi) = \begin{cases} 
  c \cdot \psi \left( \frac{x - \mu}{\xi \sigma}; \nu \right) & \text{if } x - \mu \leq 0 \\
  c \cdot \psi \left( \frac{1}{\xi} \frac{x - \mu}{\sigma}; \nu \right) & \text{if } x - \mu > 0,
\end{cases}
$$

(9)

where $\psi(z; \nu)$ is the standard Student’s $t$ distribution with degrees of freedom parameter
$\nu > 2$, $\mu$, $\sigma > 0$ and $\xi > 0$ are the location, dispersion and skewness parameters, respectively, and $c = 2\xi/(\sigma(\xi^2 + 1))$ is a constant. Because of the transformation, the parameters
cannot be interpreted directly as moments. For $\xi < 1$, the distribution is left-skewed, for
$\xi = 1$ it is symmetric, and for $\xi > 1$ it is right-skewed.

The main reason for choosing the skewed Student’s $t$ distribution lies in its flexibility.
Direct estimation of the parameters in Eqs. (A4) and (A6) would be an attractive alternative under the assumption that the model part for crashes is correctly specified.\footnote{Das and Uppal (2004) follow this approach by using GMM.}

### 3.2 Measures for conditional crash likelihood

The first dependence measure we use is based on the Gaussian copula and the empirical
copula. The Gaussian copula is related to the normal distribution and describes the
dependence implied by Wiener processes. Its bivariate functional form is given by

$$
C_2^\Phi(u, v; \rho^\Phi) = \Phi_2 \left( \Phi^{-1}(u), \Phi^{-1}(v), \rho^\Phi \right), \quad u, v \in [0, 1],
$$

(10)

where $\Phi_2()$ denotes the standard normal cumulative distribution function with correlation
coefficient $\rho^\Phi$, and $\Phi^{-1}$ denotes the inverse of the univariate standard normal cumulative
distribution function.
The empirical copula is the copula version of the empirical distribution function. For a given set of \( T \) observations \( r_i \) and \( r_m \), its functional form can be written as

\[
C_E(u, v; r_m, r_i) = \frac{1}{T} \sum_{t} I \left( r_{it} \leq r_{i}^{[uT]} \right) \cdot I \left( r_{mt} \leq r_{m}^{[vT]} \right), \quad u, v \in [0, 1],
\]

(11)

where \( r^{[uT]} \) is the \( k \)th (ascending) order statistic, \( k \) being the largest integer not exceeding \( uT \), and \( I() \) is the indicator function.

The first measure for conditional crash likelihood, which we call the empirical measure, is constructed as

\[
\lambda_{im}^{\text{emp}} = C_E(u, v; r_i, r_m) - \frac{C_2(u, v, \rho_{im}^\Phi)}{v}.
\]

(12)

The first term gives the empirical probability that the individual asset return falls below the quantile associated with probability \( u \), given that the market return lies below the \( v \)-quantile. The second term gives the same conditional probability using the Gaussian copula. If the dependence between the individual asset return and the market asset return are driven largely by diffusions, the empirical copula and the Gaussian copula will yield approximately the same result. However, if crash dependence is present, the empirical copula will give a higher joint probability than the Gaussian copula. Consequently, we interpret a high value for \( \lambda_{im}^{\text{emp}} \) as a conditional crash probability. Cappiello et al. (2005) use a similar measure to investigate comovements to identify contagion. Throughout the paper we will use \( u = v \).

The other two measures are based on the Student’s \( t \) copula. The functional form of the bivariate Student’s \( t \) copula reads

\[
C^\Psi_2(u, v; \nu^\Psi, \rho^\Psi) = \Psi_2 \left( \Psi^{-1}(u, \nu^\Psi), \Psi^{-1}(v, \nu^\Psi); \nu^\Psi, \rho^\Psi \right),
\]

\[
u^\Psi > 2,
\]

(13)

where \( \Psi_2() \) denotes the cumulative distribution function of the bivariate Student’s \( t \) distribution with correlation coefficient \( \rho^\Psi \) and degrees of freedom parameter \( \nu^\Psi \), and \( \Psi^{-1}() \) denotes the inverse of the univariate standard Student’s \( t \) cumulative distribution with degrees of freedom parameter \( \nu^\Psi \).

The main difference between the Student’s \( t \) copula and the Gaussian copula is the tail dependence that the Student’s \( t \) copula entails. Tail dependence \( \chi \) is the limit of the
conditional probability of an extreme realization of a random variable $U$, given that the realization of a random variable $V$ is extreme

$$\chi \equiv \lim_{u \downarrow 0} \frac{\Pr(U \leq u, V \leq u)}{\Pr(V \leq u)} = \lim_{u \downarrow 0} \frac{C_2(u, u)}{u}, \quad (14)$$

where $U$ and $V$ are assumed to have a marginal uniform distribution (See Joe, 1997, Sec. 2.1.10). If $\chi = 0$ the two variables do not exhibit tail dependence, and if $\chi > 0$ they do. Embrechts et al. (2002) shows that a Gaussian copula with $\rho^\Phi \neq 1$ implies tail independence. On the other hand, the Student’s $t$ copula implies tail dependence, even for $\rho^\Psi = 0$. The Student’s $t$ copula and the Gaussian copula belong to the class of elliptical copulas. The Student’s $t$ copula converges to the Gaussian copula for $\nu^\Psi \to \infty$.

Because tail dependence can be interpreted as the limit of the conditional crash probability in Eq. (7) for crashes getting more and more severe, we use it as an asymptotic measure for conditional crash probability. We base it on the Student’s $t$ copula, because the Student’s $t$ copula can also capture the elliptic dependence implied by the diffusion processes. Embrechts et al. (2002) derive a closed form expression for the tail dependence implied by the bivariate Student’s $t$ copula as

$$\lambda^a_{\nu|m} = 2 \cdot \Psi \left( \sqrt{\frac{\nu^\Psi}{\nu^\Psi + 1}} \right), \quad (15)$$

We call it the asymptotic measure for conditional crash likelihood.$^8$

The third measure we use for conditional crash likelihood is based on the degrees of freedom parameter $\nu^\Psi$ of the Student’s $t$ copula. If the conditional crash probability is low, it will not influence the actual dependence much, which will then come close to a Gaussian copula. That means that the degrees of freedom parameter estimate should be high. On the other hand, if the degrees of freedom parameter is low, the actual dependence deviates strongly from the Gaussian copula, indicating that the conditional crash likelihood is high. To give this measure the same domain as the other two measure, we define the degrees of freedom measure as

$$\lambda^\nu_{\nu|m} = 1/(\nu^\Psi - 1), \quad (16)$$

$^8$In fact, we can write $\lambda^a_{\nu|m} = \lim_{u \downarrow 0} \{C_2^\Psi(u, \nu; \nu^\Psi)/u - C_2^\Phi(u, \nu; \rho^\Psi)/u\}$, since the limit of the second term equals zero. This shows that we can interpret $\lambda^n_{\nu|m}$ as an asymptotic version of $\lambda_{\nu|m}^{\text{emp}}$, where the Student’s $t$ copula replaces the empirical copula.
which ensures $\lambda^\nu_{ijm} \in [0, 1]$. Of course, it does not have the interpretation of a probability. We use these measures for conditional crash likelihood because they complement each other. The first measure is mainly based on crash observations. However, as the number of crashes in a sample is typically low, this measure may not be very precise (i.e. have a large standard error). The other two measures are parametric, which gives the advantage that they can be estimated more precisely. However, they are based on all observations, and as a consequence they reflect other parts of the distribution as well. Since the tail dependence implied by the Student’s $t$ copula is a function of $\nu^\Psi$, the second and third measure are likely to be strongly related. However, the tail dependence is a function of $\rho$ as well, which can lead to differences. In the empirical part we investigate the relation of the three measures in more detail.

4 Crash portfolios

4.1 Data and Methods

In this section we examine portfolios constructed with the three different measures for crash likelihood. In the portfolio construction we use all regular stocks (share code 10 and 11) in the stock database of the Center for Research in Security Prices (CRSP) at the University of Chicago during the period June 1964 - November 2003. The market return series also comes from the CRSP database. The risk-free rate is the one-month Treasury bill rate from Ibbotson Associates. Both series are available on the web site of French.\(^9\)

The main step in the empirical analysis of this paper is the construction of crash portfolios. To construct portfolios that have a low, intermediate or high probability of crashing if the market crashes, we use the following approach. In each month and for each stock we calculate the values of the three crash likelihood measures presented in Section 3.2. We base these calculations on the estimates for the copula parameters over a history of 120 months (implying that stocks with a shorter history are omitted). We use the inference functions for margins method (IFM) proposed by Joe (1997) for estimation. This two step procedure first estimates the parameters for the marginal distributions, i.e. the skewed

\(^9\)http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
Student’s $t$ distributions. The second step yields the estimates for the copulas, treating the marginal distribution parameters as given. In both steps we use maximum likelihood estimators. It is possible to apply maximum likelihood estimation to jointly estimate the parameters for the marginal distributions and the copulas. While IFM is less efficient than one-step maximum likelihood estimation, it is computationally more attractive. Moreover, it guarantees that the estimates for the marginal distribution of the market return do not vary depending on which individual asset’s returns are included in the estimation.

Based on crash likelihood we sort the stocks into portfolios: portfolio L if the stock belongs to the bottom third, portfolio M if the stock falls in the middle third and portfolio H in the top third. The portfolios are value-weighted, with each stock weighted by its market value at the beginning of the month. Each portfolio contains the same number of stocks. At the end of the month the portfolio return is calculated. We also construct hedge portfolios, entailing a long position in the portfolio with the high conditional crash probability-stocks and a short position in the low conditional crash probability-stocks. We use the portfolio returns to investigate the presence of a crash risk premium. After correcting for diffusion risk, the portfolios from stocks with high conditional crash probabilities should offer a significantly positive abnormal return. On the contrary, portfolios of stocks with low conditional crash probabilities should offer insignificant abnormal returns. The average return on the hedge portfolio should be significantly positive.

Finally, we examine whether the returns on the crash portfolios can be explained by other hedge portfolio returns. We consider the size and value hedge portfolios from Fama and French (1993), a momentum portfolio as in Jegadeesh and Titman (1993), and portfolios based on coskewness as in Harvey and Siddique (2000) and cokurtosis as in Dittmar (2002). The returns on the hedge portfolios for the size, value and momentum effects are available from the web site of French (as resp. SMB, HML and UMD). We construct hedge portfolios for coskewness and cokurtosis ourselves (see Appendix B for details).

4.2 Constructing crash portfolios

The crash portfolios are constructed based on the three measures for conditional crash likelihood. Figure 1 shows the evolution of the different measures over time. It reports the
one third quantiles (solid lines) and the two thirds quantiles (dashed lines) that are used in the portfolio construction. Figure 1(a) plots the quantiles for the empirical measure. It is based on the difference between the empirical copula and the Gaussian copula. For both copulas we calculate the probability that a stock crashes given that the market crashes. A crash is defined as a return below the $u$-quantile, where we have set $u$ equal to 5%. If the empirical and the Gaussian copulas do not differ significantly from each other, the empirical measure will be close to zero. We observe that the one third quantile is close to zero. Sometimes, it lies below zero, indicating that for some stocks the Gaussian copula implies a higher conditional probability of returns below the 5% quantile than empirically observed. For many stocks the empirical conditional probability of returns below the 5% quantile exceeds the Gaussian implied probability. For one third of the stocks this difference is easily 0.15 or larger. We will see later whether these stocks offer on average higher returns.

[Figure 1 about here.]

The asymptotic measure in Figure 1(b) and the degrees of freedom measure in Figure 1(c) are based on the Student’s $t$ copula. Because both measures are functions of the degrees of freedom parameter, the patterns of the quantiles show a clear resemblance. For the one third and the two thirds quantiles the correlation coefficients equal 0.69 and 0.82, respectively. We clearly see the effect of the crash of October 1987 as both measures show an immediate increase after it. However, the figures indicate that the crash of October 1987 ends a short period of low conditional crash probabilities, as it seems that the evolution of the measures over 1988-1997 is a continuation of 1975-1985. An inspection of the data shows that July, August and September of 1974 were notorious crash months with market returns as low as -7.79%, -9.37% and -11.78% (and a rebounce of 16.05% in October 1974). Because these months drop from the estimation horizon from July 1984 onwards, we see a decrease towards the beginning of 1985. At the end of 1997 we see a similar decrease in the conditional crash probability measures. By the end of the 1990s the measures revert partially. The time trend that seems to be present in the two thirds quantile of the asymptotic measure does not show up in the two thirds quantile of the degrees of freedom measure.
All measures exhibit considerable time variation, justifying our preference for the portfolio approach combined with a rolling estimation window. In each subfigure, the solid and dashed lines follow a similar pattern, indicating that changes in conditional crash likelihood are similar across stocks (the correlations between the one third quantiles and two thirds quantiles are 0.96 for the empirical measure, 0.85 for the asymptotic measure and 0.94 for the degrees of freedom measure). The empirical measure and the other two measures do not seem to be much related. Considering the one third quantiles we find correlation coefficients of 0.29 between the empirical measure and the asymptotic measure, and 0.44 between the empirical measure and the degrees of freedom measure. For the two thirds quantiles correlation coefficients are 0.24 and 0.37 respectively.

A first glance on the different portfolios that we construct based on the conditional crash likelihood measures is provided in Table 1. For each measure, we show the characteristics of the three portfolios, portfolio L with the stocks that have the lowest values for that measure (below the one third quantile), portfolio M with stock that fall in intermediate range (between the one third and two thirds quantile), and portfolio H with stocks in the top third quantile. The average values for the one third quantile are all close to zero (0.026 for the empirical measure, 0.017 for the asymptotic measure and 0.050 for the degrees of freedom measure). The two thirds quantiles equal on average 0.162, 0.124 and 0.165, respectively, and Figure 1 shows that they are larger than zero for each month. We interpret the relatively low values for the two third quantiles as an indication that idiosyncratic shocks account for a large proportion of the crashes in individual stocks. However, given the investor’s general aversion to market wide losses, the premium can still have a considerable impact on expected returns.

[Table 1 about here.]

The average excess returns that we find for the different portfolios point in the direction of a reward for crash risk. The average excess returns on the L portfolios are considerably lower than the returns on the M and H portfolios. The difference varies from 1.63% to 2.74% on an annual basis. Moreover, in case of the asymptotic measure and the degrees of freedom measure, the volatility of the L portfolios exceeds the volatility of the M and H portfolios. Consequently, the Sharpe ratios for the L portfolios are considerably less
attractive than for the M and H portfolios. Gathering all stocks into one value-weighted portfolio produces a Sharpe ratio of 0.54 over the period June 1974 - November 2003.\footnote{The Sharpe ratio for the complete market over this period equals 0.43, which is considerably lower. However, the portfolios we construct contains only stocks with a history of more than 10 years, which means that all stocks that have not been listed for 10 years are excluded.} Of course, we have not taken differences in standard market risk exposure into account. In the next subsection, we check whether the M and H portfolios outperform the L portfolios after a correction for market risk. We also check whether the return series are related to other risk factors.

### 4.3 Crash portfolio analysis

In this subsection we put the different portfolios constructed in the previous subsection under further scrutiny. The first results in Table 1 indicate that the M and H portfolios perform better than the L portfolios. However, this difference may be caused by different exposures to market (diffusion) risk. In this subsection we correct for this exposure, and examine whether the pattern of Table 1 remains. By constructing a hedge portfolio we test whether a long position in stocks with high conditional crash probabilities and a short position in stocks with low conditional crash probabilities yields a positive average pay-off.

It is also possible that the better performance of the M and H portfolios can be explained by other trading strategies that yield significant outperformance. We consider the familiar strategies based on size and value versus growth, as proposed by Fama and French (1993, 1995) and momentum as put forward by Jegadeesh and Titman (1993). Harvey and Siddique (2000) show that a Taylor expansion of the pricing kernel leads to the inclusion of coskewness and Dittmar (2002) extends this to cokurtosis. Under regular assumptions on utility functions, investors have a preference for increasing coskewness and decreasing cokurtosis. We construct coskewness and cokurtosis hedge portfolios and examine whether the crash portfolios are related to these portfolios. In appendix B we examine the trading strategies in more detail and discuss the construction of the coskewness and cokurtosis portfolios. We include two sets of coskewness and cokurtosis portfolios in our analysis. The first set is constructed similarly as the crash portfolios, using a 120-months estimation
window. This set can indicate whether our crash measures capture to a large degree the same information as coskewness and cokurtosis measures would. The second set uses an estimation window of 60 months, and more resembles the approach of Harvey and Siddique (2000).

Table 2 shows the regression results for the portfolios constructed with the empirical measure for conditional crash likelihood. Panels (a) to (c) consider the portfolios $L^{\text{emp}}, M^{\text{emp}}$ and $H^{\text{emp}}$ and panel (d) reports the results on the hedge portfolio. We conduct simple OLS regression and calculate Newey-West standard errors. The coefficients on $r_m$ indicate that the $L^{\text{emp}}, M^{\text{emp}}$ and $H^{\text{emp}}$ portfolios all exhibit significant exposures to the market return. After correcting for this exposure, the $M^{\text{emp}}$ and $H^{\text{emp}}$ portfolios still show significant $\alpha$’s of 0.26% (portfolio $M^{\text{emp}}$) and 0.33% (portfolio $H^{\text{emp}}$) per month. The $\alpha$ for the $L^{\text{emp}}$ portfolio is insignificant. We interpret this result as evidence that a portfolio with a higher crash exposure offers an extra return that cannot be explained by exposure to the market. The actual return due to exposure to market crashes may be even higher, because the estimated coefficient on $r_m$ will capture crash exposure for a small part.

[Table 2 about here.]

The portfolios have only limited exposure to other trading strategies. The coefficients on the size, value or momentum portfolios are significant in a few cases only, and the coefficients are generally small. The small negative coefficients on SMB may indicate that our selection procedure is slightly biased towards big firms. The relation with coskewness is stronger. The portfolios $M^{\text{emp}}$ and $H^{\text{emp}}$, which have a relatively large exposure to crash risk, have significant coefficients on the coskewness hedge portfolios. The coefficient on the 120-months coskewness portfolio is larger than the coefficient on the 60-month coskewness portfolio. However, since the 120-month coskewness portfolio does not yield a significant positive return (see Table 7), the $\alpha$’s of the crash portfolios are not much affected. If we use the 60-month coskewness portfolio, which yields a significant return, the $\alpha$’s of the $M^{\text{emp}}$ and $H^{\text{emp}}$ portfolios decrease but remain significant at the 5% level.\footnote{Including both the 120 months and the 60 months coskewness portfolios in one regression, shows that the explanatory effect of the 60 month portfolio is completely captured by the 120 month portfolio.} The $L^{\text{emp}}$ and $H^{\text{emp}}$ portfolios show significantly negative exposures to the cokurtosis portfolios. Because the
return on the kurtosis portfolio is insignificant, the α’s of the crash portfolios do no change much. Depending on the risk corrections in the regressions, the portfolio with stocks with the largest exposure to crash risk yield an extra return of 2.5% to 4% on an annual basis.

In panel (d), we report the results on the hedge portfolio, constructed by a long position in portfolio $H^{emp}$ and a short position in portfolio $L^{emp}$. The average return on this portfolio equals 0.23% per month and is significant at the 5% level. This hedge portfolio does not have a significant exposure to the market return, nor to the size, value or momentum portfolios. Since portfolio $H^{emp}$ has an exposure to the coskewness factor, whereas portfolio $L^{emp}$ does not, the hedge portfolio shows a similar exposure to the coskewness factor as portfolio $H^{emp}$. A correction for this exposure based on 120-month coskewness portfolio does not affect α much as it remains significant at 0.20% per month. This entails a yearly outperformance of 2.5%. If the 60-month coskewness portfolio is used, the α decreases but remains marginally significant.

In Table 3 we consider the portfolio constructed with the asymptotic measure. The $L^a$ portfolio contains stocks with hardly any tail dependence with the market (the average upper bound for this portfolio is reported in Table 1 as 0.017). Zero tail dependence is consistent with dependence completely driven by the diffusion processes. If we correct for the correlation with the market return, the abnormal return of the $L^a$ portfolio is positive but not significantly different from zero. The $M^a$ and $H^a$ portfolios show dependence that cannot stem from diffusion processes as discussed in the theoretical section. Panels b and c of Table 3 show that the α’s of the $M^a$ and $H^a$ portfolios remain significantly positive after correcting for market risk. Strangely, the α of the $M^a$ portfolio exceeds the α of the $H^a$ portfolio, though not significantly.

[Table 3 about here.]

Including the hedge portfolios SMB, HML and UMD in the regressions yields similar estimates as in Table 2. The coefficients are small and mostly insignificant. Each portfolio has a significant sensitivity to the coskewness factor of about the same size, which is

\[12\] In case of the empirical measure, both positive and negative deviations from the Gaussian copula are possible. This means that the $L^{emp}$ portfolio cannot be related directly to no deviations from the Gaussian copula, contrary to $L^a$ portfolio.

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a bit puzzling. Apparently, selecting stocks on their tail dependence with the market is different from selecting stocks on coskewness. Sensitivities to cokurtosis are different across portfolios. The L\textsuperscript{a} portfolio has a significant negative exposure to the cokurtosis hedge portfolio, indicating that it is more platykurtic. We find that H\textsuperscript{a} portfolio is more leptokurtic, as it has a positive sensitivity to the 60-months cokurtosis portfolio. Since tail dependence is an asymptotic concept, it should reflect the joint behavior in the extreme parts of the distribution. Consistent with this statement, the sensitivities of the portfolios in Table 3 compared with those in Table 2 show that cokurtosis gains importance while coskewness looses.

The results on the hedge portfolio in panel (d) of Table 3 indicate that shorting assets with hardly any tail dependence and investing in assets with relatively high tail dependence does not yield a significant abnormal return, though it is on average positive. The hedge portfolio inherits the sensitivity to cokurtosis from the L portfolio.

Constructing portfolios with the degrees of freedom measure for conditional crash likelihood is almost the same as using the asymptotic measure. The results in Table 4 are virtually the same as in Table 3. The returns to the hedge portfolio are somewhat larger, but remain insignificant. The correlation coefficient of the portfolios constructed with the degrees of freedom measure with their respective companion portfolios from the asymptotic measure are all larger than 0.99. The hedge portfolios are also highly correlated.

Based on this empirical analysis we conclude that a crash risk premium is present in the cross section of stock returns. Buying stocks that score highly on the measures that we have constructed yields an extra yearly return of 2\% to 4\% after correction for other risk exposures. However, only the empirical measure leads to a profitable hedge portfolio with a statistically significant extra return of 2.5\% per year. This hedge portfolio is almost completely uncorrelated with the hedge portfolios of the asymptotic and the degrees of freedom measures, indicating that these measures are complements and not substitutes.
5 Explaining the cross section of stock returns

The question that remains to be answered is whether crash risk can contribute to explaining the cross section of stock returns. In the previous section we established that portfolios with a relatively high exposure to crash risk earn on average an extra pay-off after correcting for exposure to market risk and possible other risk factors, pointing at the presence of a crash risk premium. In this section we examine whether a combination of a market diffusion factor and a market crash factor leads to insignificant pricing errors. We start with an investigation of the explanatory power of the crash risk portfolio on a set of well-known portfolios. Then we consider the cross section of individual stock returns.

5.1 Portfolios tests

In its search for explanations for the cross section of stock returns, empirical research has established several groups of portfolios whose return differences could not be explained sufficiently by the traditional CAPM. From these portfolios the hedge portfolios are constructed that are used many times as a risk factor, i.e. the size, value and momentum factors. While these portfolios are specifically constructed for asset pricing tests, industry portfolios are also often used as they suffer less from data snooping.

In this subsection we consider portfolio sets based on industries, size, value and momentum. For each set we conduct a cross sectional test as described in Cochrane (2001, Ch. 12). We estimate the exposure to risk factors for each portfolio $i$ in a time series regression

$$R_{it} = a_i + \beta_i' f_t + e_{it},$$

(17)

where $R_{it}$ is the excess return on portfolio $i$, $f_t$ is a vector with the factor values, i.e. excess returns, at time $t$, and $\beta$ is a vector with sensitivities. We use these sensitivities to estimate the risk premia $\zeta$ for the factors:

$$E[R_{it}] = \alpha_i + \zeta' \beta_i.$$  

(18)

The term $\alpha_i$ has the interpretation of a pricing error. If the risk factors $f$ accurately explain the cross section of stocks returns, the pricing errors should be zero. We estimate
the model in Eqs. (17) and (18) in a GMM framework. In this way we automatically include the Shanken (1992) correction in the covariance matrix of the pricing errors for the fact that the sensitivities are estimated. Moreover, by using the weighting scheme from Newey and West (1987) for the spectral density matrix, we can incorporate autocorrelation and heteroskedasticity. Since Eq. (18) implies a number of moments equal to the number of portfolios and only one parameter (the risk premium) per risk factor, we weigh the moments by $\beta$ to construct the GMM objective, as discussed in Cochrane (2001, Ch. 12). In this setup we can use the $TJ_T$ statistic to test for significant pricing errors. Under the null hypothesis of zero pricing errors, this statistic follows a $\chi^2$-distribution with degrees of freedom equal to the number of portfolios minus the number of risk premia.

The data for the industry, size and value portfolios are based on the CRSP database and are available on the web site of French. The industry set consists of 10 portfolios based on SIC codes: Consumer Non-Durables (1), Consumer Durables (2), Manufacturing (3), Energy (4), High Tech (5), Telecom (6), Shops (7), Health (8), Utilities (9) and Others (10). The size portfolios are 10 portfolios with stocks sorted on market equity from small (1) to large (10). The value portfolios are 10 portfolios with stocks sorted on the book-to-market ratio, from low (growth, 1) to high (value, 10). The set of momentum portfolios is based on the CRSP database and can be downloaded from the website of Van Vliet. This set consists of 10 portfolios with stocks sorted on their performance over the one to twelve months prior to portfolio formation (see Post and van Vliet, 2004). The industry, size and value portfolios are available over the entire period for which we have constructed crash risk portfolios (June 1974 - November 2003). The momentum portfolios end at December 2002, implying a slightly shorter horizon for the test based on the momentum portfolios.

For each set of portfolios, we conduct three cross-sectional tests. First, we conduct a test of the traditional CAPM. In the second test we include the hedge portfolio based on the empirical measure for conditional crash likelihood. In the third test we include the excess returns on the $H_a$ portfolio, which is based on the asymptotic measure for conditional crash likelihood. We do not include the hedge portfolio based on the asymptotic measure for

\[ \text{See the web site of Kenneth French, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, for detailed information on portfolio construction.} \]

\[ \text{See http://www.few.eur.nl/few/people/wvanvliet/datacenter/index.htm.} \]
two reasons. First, the exposure to crash risk should be concentrated completely in the $H^{texta}$ portfolio, when using the asymptotic measure. For the empirical measure this does not apply by definition. Second, the hedge portfolio did not yield a significant expected positive pay-off, whereas the $H^{texta}$ portfolio did. We do not consider the portfolios based on the degrees of freedom measure, because they are to a large extent identical to the portfolios based on the asymptotic measure.

An alternative approach would be to use the fact that all factors we consider are returns themselves and conduct a time series test (i.e. test whether the $a_i$’s in Eq. (17) are insignificant). The factor risk premium is then taken equal to the average factor value. However, we have a specific reason not to use this approach. We want to establish two risk factors: a market diffusion factor and a market crash factor. As they are both present in the market return, we cannot estimate the market diffusion premium as the time series average of the market return. In a cross-sectional approach we do not have to make such an assumption.

Table 5 shows the results from the cross-sectional tests on the different portfolios. It reports the estimated risk premia and the pricing errors, together with their standard errors, and the $TJ_T$-statistic with a $p$-value. We draw several conclusions based on this table. First, the addition of the $H^a$ portfolio improves the explanatory power of the model. Generally, we see a decrease of the $TJ_T$ statistic and an increase of the $p$-value. This improvement is most notable for the momentum portfolios (see panel d). The traditional CAPM is rejected for momentum portfolios, but after the addition of the $H^a$ portfolio, the pricing errors are not jointly significant anymore. In the appendix we show that the momentum effect is by far larger than the size and the value effect. In case of the traditional CAPM, six out of ten portfolios exhibit significant pricing errors. Not all pricing errors vanish, but they become smaller and less significant.

[Table 5 about here.]

Second, the hedge portfolio based on the empirical measure for conditional crash likelihood performs worse than the $H^{texta}$ portfolios. While we generally see a decrease of the $TJ_T$ statistic and an increase of the $p$-value, we also observe large standard errors for the
estimates of the pricing errors. Addition the $H^a$ portfolio leads to more precise estimates of the pricing errors than the addition of the hedge portfolio based on the empirical measure.

Third, we observe that the estimates for the risk premium associated with the $H^a$ portfolio are positive and significant, with exception of the case of the size portfolios. Moreover, in the other three cases the premium is considerable and exceeds the premium for market risk. The premium on the hedge portfolio based on the empirical measure is not significant. The size of this premium stresses the importance of crash risk.

5.2 Individual stocks

We conclude by examining the added value of a crash risk factor on the entire cross section of stock returns instead of several portfolios as in the previous subsection. Crucial for a correct estimation of a risk premium (and hence the pricing errors) is enough variation in the sensitivities for the risk factors. Considering the entire cross section yields the largest possible variation. On the other hand, a direct test of the hypothesis that the pricing errors are jointly zero on each stock cannot be conducted, since too few observations are available to estimate the covariance matrix of the pricing errors. Therefore, we present the results in this subsection as complementary to the results in the previous subsection.

We conduct this analysis in a Fama and MacBeth (1973)-framework to allow changes in the sensitivities over time. We estimate the time series regression in Eq. (17) over 60 months. For month 61 we estimate the cross-sectional regression

$$R_{ei} = \alpha_i + \zeta' \beta_i, \quad (19)$$

where $R_{ei}$ is the excess return for stock $i$ for month 61, $\beta_i$ is the vector with sensitivities estimated over the prior 60 months, $\alpha_i$ is the pricing error and $\zeta$ is the vector of risk premia. Based on the outcomes we calculate the cross-sectional average pricing error. To be included in this regression for a certain month, a stock needs to have a complete return series over 61 months. We start with the month 1 to 60 for the time series regression and 61 for the cross-sectional regression, then we consider month 2 to 61 for the time series regression and 62 for the cross-sectional regression, and so on. After this procedure is ended, we have a time series for each risk premium and for the average pricing error.
Based on these series we calculate the time series average of the risk premia and the time series average of the (average) pricing error, and the corresponding standard error.

Table 6 presents the results of this analysis. We report two estimates for standard errors: an estimate that is only based on the time series of premia and pricing errors, and another estimate that includes corrections due to Shanken (1992) for the fact that the sensitivities $\beta_i$ are estimated as discussed in Cochrane (2001, Ch. 12). We find that the CAPM is rejected, as it leads to significant pricing errors of 0.51% per month. Adding the empirically based hedge portfolio is an improvement, since the average pricing error decreases. However, the premium for this factor is not significant. Adding the $H^{extra}$ portfolio leads to a further reduction of the average pricing error to 0.26% per month. The estimated standard errors decrease as well, and consequently the average pricing error is still significant. The premium for this crash risk factor is 0.71% per month and is significant at the 1% level for the time series standard error and at the 5% level if Shanken (1992)-corrections are included.

Combining the results of this analysis with those of the previous subsection, we conclude that crash risk contributes to explaining the cross section of stock returns. For the asymptotic measure we find lower pricing errors and lower corresponding standard errors. The momentum effect, which is the largest compared to the size and value effects (see Table 7), can be explained by crash risk. The pricing errors associated with momentum portfolios are not significant anymore after the addition of a crash risk factor. Moreover, the $H$ portfolio based on the asymptotic measure leads to a significant premium. The estimates for this premium vary based on the techniques and samples, but confidence intervals of the premium estimate all include the 0.71% that is estimated based on the broadest cross section. This would put crash risk on equal importance as market diffusion risk.

6 Conclusion

We have investigated whether crash risk is present in the cross section of stock returns. First, we have derived an extension of the traditional CAPM that includes crash risk. In
this extension of existing asset pricing models that capture crash risk like Merton (1971), Ho et al. (1996) and Bates (2001), we allow assets to vary in their likelihood to crash, given that the market crashes. It states that each asset should pay a premium for crash risk that is the product of the market crash risk premium and the sensitivity that the asset exhibits with regard to market crashes. This premium is potentially large, depending on the investors’ crash aversion, the likelihood of a crash and the size of a crash.

We have reported evidence for a crash risk premium in the cross section of US stock returns over the period 1964 to 2004. This evidence is based on a sorting stocks based on their crash likelihood, given that the market crashes. We have derived three measures to determine this likelihood. For all three measures, the portfolio with stocks that score high on a measure pay on average a significant extra pay-off after correcting for traditional market risk. This premium varies between 2.4% and 4% depending on which measure is used. This return cannot be explained by other risk factors. Stock that score low on the crash likelihood measures do not pay a significant extra premium. The estimates for a crash risk premium based on cross-sectional regressions is even larger at 8.4%, putting crash risk at the same level as traditional market diffusion risk. Consequently, crash risk may be more important than previously thought. Apparently, investors exhibit show considerable crash aversion on top of general risk aversion.

We conclude that a crash risk factor can contribute to the explanation of the cross-sectional variation of stock returns. The return differences in momentum portfolios, which cannot be explained by the traditional CAPM, vanish to a large extent when we add a crash factor to the CAPM. Moreover, we find that the crash portfolios are related to the coskewness and cokurtosis factors. This evidence may indicate that the previously reported momentum, coskewness and cokurtosis effects are actually rewards for a higher exposure to crash risk. Together, these findings call for a further investigation of the effects of crash risk on asset pricing.
A Derivation of the crash-CAPM

In this appendix we derive the extended version of the CAPM that takes crash risk into account, based on Bates (2001). We show how the CAPM arises in an endowment economy with a representative agent who consumes at a final date, $T$. The assumption underlying this model are stated in Section 2. Before we start we state an extended version of the lemma in Bates (2001, p. 12) that we use in the derivation.

**Lemma** Let $Y_{1t}$ and $Y_{2t}$ be two random variables that follow stochastic processes

$$dY_{it} = \mu_i dt + \sigma_i dZ_i, \ i = 1, 2,$$

where $dZ_1$ and $dZ_2$ are correlated Wiener processes with $E[dZ_1 dZ_2] = \rho$. Let $N_t = (N_{1t} \ N_{2t})'$ be a bivariate Poisson process that evolves according to

$$dN = \begin{cases} 
(1 \ 1)' & \text{with probability } \lambda_{11} \ dt \\
(1 \ 0)' & \text{with probability } \lambda_{10} \ dt \\
(0 \ 1)' & \text{with probability } \lambda_{01} \ dt \\
(0 \ 0)' & \text{with probability } \lambda_{00} \ dt,
\end{cases}$$

where the arrival rates are larger than or equal to zero and sum to one. The Wiener processes and Poisson processes are independent.

The expectation of a function $F(Y_{1t}, Y_{2t}, N_{1t}, N_{2t}) = \exp (c_1 Y_{1t} + c_2 Y_{2t} + d_1 N_{1t} + d_2 N_{2t})$, with $c_1$, $c_2$, $d_1$ and $d_2$ deterministic constants, can then be found as

$$E_t [F(Y_{1T}, Y_{2T}, N_{1T}, N_{2T})] = F(Y_{1t}, Y_{2t}, N_{1t}, N_{2t}) \cdot \exp \left( \left( c_1 \mu_1 + c_2 \mu_2 + \frac{1}{2} c_1^2 \sigma_1^2 + c_1 \sigma_1 \rho c_2 \sigma_2 + \frac{1}{2} c_2^2 \sigma_2^2 + \lambda_{11} (e^{d_1 + d_2} - 1) + \lambda_{10} (e^{d_1} - 1) + \lambda_{01} (e^{d_2} - 1) \right) (T - t) \right).$$

**Proof** Use Itô’s lemma to derive

$$E_t [dF_t] = F_t \left( c_1 \mu_1 + c_2 \mu_2 + \frac{1}{2} c_1^2 \sigma_1^2 + c_1 \sigma_1 \rho c_2 \sigma_2 + \frac{1}{2} c_2^2 \sigma_2^2 + \lambda_{11} (e^{d_1 + d_2} - 1) + \lambda_{10} (e^{d_1} - 1) + \lambda_{01} (e^{d_2} - 1) \right) dt,$$

and use the standard solution for the partial differential equation. ■
First we derive the pricing kernel. In equilibrium the market clears and the representative agent holds the market claim. His marginal utility as time $T$ can be found by differentiation Eq. (3)

$$
\eta_T \equiv \partial W U(W_T, N_{mT}, T)\big|_{W_T=X_{mT}} = e^{\delta N_T} X_T^{-\gamma}.
$$

(A1)

Consequently, a pricing kernel that is valid at time $t$ would be $\eta_T/\eta_t$.

We can use this pricing kernel to price assets. Since we are interested in excess returns, we use the riskless asset as numeraire. As a consequence, $\eta_t = E_t[\eta_T]$. Using the lemma we find

$$
\eta_t = E_t[\eta_T] = e^{\delta N_t} X_t^{-\gamma} \exp\left\{ \left( -\gamma \mu_m + \frac{1}{2} \gamma^2 \sigma_m^2 + \lambda_m (e^{\delta - \gamma \kappa_m} - 1) \right) (T-t) \right\}.
$$

(A2)

The price $S_{mt}$ of the market claim satisfies $\eta_t S_{mt} = E_t[\eta_T X_{mT}]$ and based on the lemma we derive

$$
S_{mt} = X_{mt} \exp \left\{ \mu_m + \frac{1}{2} \sigma_m^2 - \gamma \sigma_m^2 + \lambda_m e^{\delta - \gamma \kappa_m} (e^{\kappa_m} - 1) (T-t) \right\}
$$

(A3)

Applying Itô's lemma yields the process followed by the market asset

$$
\frac{dS_m}{S_m} = \left( \gamma \sigma_m^2 - \lambda_m e^{\delta - \gamma \kappa_m} (e^{\kappa_m} - 1) \right) dt + \sigma_m dZ_m + (e^{\kappa_m} - 1) dN_m
$$

(T-A4)

Taking expectation produces the expected return on the market asset in (6).

The price of the each individual asset $i$ satisfies the fundamental relation $\eta_t S_{it} = E_t[\eta_T X_{iT}]$ as well. Apply the lemma to find

$$
S_{it} = X_{it} \exp \left\{ \left( \mu_i + \frac{1}{2} \sigma_i^2 - \gamma \sigma_i \rho_{i,m} \sigma_m + \left( \lambda_m - \lambda_{im} \right) (e^{\delta - \gamma \kappa_m} - 1) + \lambda_{im} \left( e^{\delta - \gamma \kappa_{im} + \gamma \kappa_i} - 1 \right) \right) (T-t) \right\}
$$

(A5)

The process for the asset can then be derived by applying Itô’s lemma

$$
\frac{dS_i}{S_i} = \left( \gamma \sigma_i \rho_{i,m} \sigma_m - \left( \lambda_i + \lambda_{im} \left( e^{\delta - \gamma \kappa_m} - 1 \right) \right) (e^{\kappa_i} - 1) \right) dt + \sigma_i dZ_i + (e^{\kappa_i} - 1) dN_i
$$

(A6)

and taking expectations produces the expected return in Eq. (7).
B Trading strategies

In this appendix we discuss the hedge portfolios used in Section 4.3. Financial researchers have established several investment strategies that yield a positive significant abnormal return that cannot be explained by exposure to other risk factors. These strategies generally consist of constructing hedge portfolios: buy assets that score highly on a certain measure and sell assets that score lowly on it.

The two best-known strategies are due to Fama and French (1993, 1995). Their first strategy exploits the small firm effect by buying stocks of small firms and selling stocks of big firms. The resulting portfolio is commonly referred to as SMB (Small Minus Big). The second entails buying value stocks (firms having a high value for the ratio of book equity to market equity) and selling those of growth stocks (firms with a low book-to-market ratio). It is often denoted as HML (High Minus Low). French’s web site provides returns on both hedge portfolio based on the CRSP database from 1926 onwards. Over the sample period that we consider in this paper, the returns on the SMB and the HML portfolios add up to 3.36% and 5.13% per year, respectively.

Jegadeesh and Titman (1993) show that buying stocks that did relatively well in the recent past (based on a history of three up to twelve months) and selling stocks that performed relatively poorly in the recent past yields a profit. The UMD (Up Minus Down) portfolio, available on French’s web site, is based on this strategy. Over the period June 1974 - November 2003, the average return on this portfolio equals 10.67% per year.

The other strategies we consider are based on higher order extensions of the CAPM. One of the possible explanations for the outperformance of the size, value and momentum portfolios argues that these portfolios capture non-linearities in the pricing kernel. Under the assumption that the market portfolio is an accurate proxy of the wealth portfolio, this means that the expected return on a specific stock is a non-linear function of the market return. Harvey and Siddique (2000) derive an extension of the CAPM in which the pricing kernel is a function of the market return and the squared market return. They show that their model implies that coskewness is priced. Coskewness measures to which degree an asset return increases when the squared market return increases. Because standard utility theory prescribes decreasing absolute risk aversion for increasing wealth (see Arditti, 1967),
stocks with negative coskewness should pay a premium. Dittmar (2002) carries the extension of Harvey and Siddique (2000) one step further by adding the cubic market return to the pricing kernel equation. This introduces cokurtosis as a factor determining the expected return of an asset. Positive cokurtosis indicates that an asset moves in the same direction as the cubic market return. Under the assumption of decreasing absolute prudence (see Kimball, 1993), investors require a premium for stocks with positive cokurtosis.

As standard accepted portfolios are not available for coskewness and cokurtosis, we construct hedge portfolios capturing their effects. We follow the same approach as we did for the crash portfolios (see Section 4.1), meaning that we start by estimating coskewness and cokurtosis over 120 months. We follow Harvey and Siddique (2000) and construct a coskewness measure $\beta_{SKD}^{i,t}$ as

$$\beta_{SKD}^{i,t} = \frac{E_t [\epsilon_{i,t+1}^2 \epsilon_{m,t+1}^2]}{\sqrt{E_t [\epsilon_{i,t+1}^2] E_t [\epsilon_{m,t+1}^2]}},$$

where $\epsilon_{i,t+1}$ is the abnormal return on stock $i$ at time $t$ that results from the standard CAPM, and $\epsilon_{m,t+1}$ is the abnormal return on the market. We estimate this measure based on the residuals of the CAPM regression for asset $i$ over the previous 120 months and the market returns minus their 120 months average. Because Dittmar (2002) does not define a measure for cokurtosis, we define a cokurtosis measure $\beta_{KTD}^{i,t}$ similar to the measure for coskewness

$$\beta_{KTD}^{i,t} = \frac{E_t [\epsilon_{i,t+1}^3 \epsilon_{m,t+1}^2]}{\sqrt{E_t [\epsilon_{i,t+1}^2] (E_t [\epsilon_{m,t+1}^2])^{3/2}}}. \quad (B2)$$

Based on these measures we construct three equally sized, value weighted portfolios, and use next month’s stock returns to calculate portfolio returns. Finally, we construct a hedge portfolio for coskewness by a long position in the portfolio with stocks that have low values for the coskewness measure and a short position in portfolio with stocks that score highly. The average return on this NMP (Negative Minus Positive) portfolio equals 1.05% per annum. For cokurtosis we construct a hedge portfolio by a long position in the portfolio of assets with high cokurtosis and a short position in the low cokurtosis portfolio. For this LMP (Leptokurtic Minus Platykurtic) portfolio we find an average return of -0.72%.

The returns we find on the coskewness portfolio is low compared to the returns on the other hedge portfolios. Harvey and Siddique (2000) report a return on their coskewness
hedge portfolio of 3.60% per annum. However, they estimate the coskewness measure in Eq. (B1) over 60 months and calculate the average return over the period July 1963 - December 1993. If we select the same stocks as in the 120-month setup but use an estimation window of 60 months, we end up with an average return of 3.60% per month.\textsuperscript{15} The correlation between the 60-months and 120-months portfolio equals 0.66. Therefore, we will consider this hedge portfolio as well.

The return on the cokurtosis hedge portfolio is also small and has the wrong sign. In this case, the consequences of changing the estimation window are small, as the average return based on a 60 month estimation window equals -0.93%. Dittmar (2002) remarks that the inclusion of human capital into the wealth portfolio is crucial for determining the risk premium on kurtosis. The absence of human capital in our wealth portfolio may explain the low value and wrong sign of the result on the kurtosis portfolio. Including human capital is beyond the scope of this research. Another explanation can be that the measure for cokurtosis in Eq. (B2) is not accurate. However, if we use the regression coefficient of the stock return on the cubic market return (similarly, Harvey and Siddique, 2000, use the regression coefficient of the stock return on the squared market return as alternative measure for coskewness) or the measures proposed by Christie-David and Chaudhry (2001)\textsuperscript{16}, we find similar results.

In Table 7 we report the average abnormal return ($\alpha$) of the market return and the different hedge portfolios, after correcting for exposure to market risk and other hedge portfolios. We find that the average return on the market is 0.59% per month or 7.06% per annum. Of the two Fama and French-factors, only HML yields a significantly positive $\alpha$ over the period June 1974 - November 2003 of 7.45% per year. The momentum factor is not much affected by exposure to market risk or the size and the value effect, and remains high at a yearly abnormal return of 12.50%. Of the coskewness and cokurtosis hedge portfolios only the coskewness portfolio based on a 60 month estimation window shows a (marginally) significant $\alpha$, that equals 3.28% on a yearly basis.

\textsuperscript{15}If we include all stocks for which a 60 month history is available, we find an average return of 3.71%. This indicates that the estimation window is crucial to forming hedge portfolios based on coskewness.

\textsuperscript{16}They propose to measure cokurtosis as $E_t[R_{it+1}\epsilon_{m,t+1}]/E_t[\epsilon_{m,t+1}^4]$, where $\epsilon_{m,t+1}$ is the abnormal return on the market.
[Table 7 about here.]
References


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This table presents summary statistics of the different portfolios into which the stocks are sorted based on a measure for its conditional crash probability, given that the market crashes. We use the three different measures for this probability and calculate the value of these measures for each stock in each month as described in the caption of Figure 1. For each month we construct three value-weighted portfolios with an equal number of stocks, based on the values of the measures for the conditional crash probability, using the market values at the beginning of the month as weights. We label the portfolio L, M and H reflecting low, intermediate and high values of the conditional crash likelihood measure. For each portfolio and each measure we report the average lower and upper bound used to construct the portfolio ($\lambda_{L}^{emp}$ and $\lambda_{H}^{emp}$), the average excess return $\mu$ (in % per annum), the volatility $\sigma$ (in % per annum) and the annual Sharpe ratio $\mu/\sigma$. 

|                | sorted on $\lambda^{emp}_{\ell|m}$ | sorted on $\lambda^{a}_{\ell|m}$ | sorted on $\lambda^{\nu}_{\ell|m}$ |
|----------------|-----------------------------------|-----------------------------------|-----------------------------------|
|                | $L^{emp}$ $M^{emp}$ $H^{emp}$      | $L^{a}$ $M^{a}$ $H^{a}$          | $L^{\nu}$ $M^{\nu}$ $H^{\nu}$   |
| $\lambda_{L}$  | -1                                | 0                                 | 0                                 |
| $\lambda_{U}$  | 0.026                              | 0.16                              | 0.12                              |
| $\mu$          | 5.92                               | 7.98                              | 8.66                              |
| $\sigma$       | 13.23                              | 13.46                              | 14.11                             |
| $\mu/\sigma$   | 0.45                               | 0.59                               | 0.61                              |

Table 1: Portfolio statistics for different sorts
Table 2: Analysis of portfolios constructed based on the empirical conditional crash likelihood measure.

(a) portfolio L$^{\text{emp}}$ (bottom third)

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$r_m$</th>
<th>SMB</th>
<th>HML</th>
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<td>0.64$^a$</td>
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<td>0.11$^b$</td>
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(b) portfolio M$^{\text{emp}}$ (middle third)

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(c) portfolio H$^{\text{emp}}$ (top third)

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(d) hedge portfolio (portfolio H$^{\text{emp}}$ - portfolio L$^{\text{emp}}$)

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This table present the regression results for different regressions of the monthly returns of the L$^{\text{emp}}$, M$^{\text{emp}}$ and H$^{\text{emp}}$ portfolios on a constant, the excess market return and other factor portfolios. The L$^{\text{emp}}$, M$^{\text{emp}}$ and H$^{\text{emp}}$ portfolios are value weighted portfolios with equal numbers of stocks, constructed based on the empirical measure for conditional crash likelihood (see Eq. (12)) for the different stocks, estimated over the preceding 120 months. As regressors we consider the excess market return ($r_m$), the size factor (SMB), value factor (HML) and momentum factor (UMD) available on the website of French, and a coskewness factor (NMP) and cokurtosis factor (LMP) (both constructed as described in Appendix B, based on an estimation window of 120 months and of 60 months (labeled NMP60 and LMP60)). Newey-West standard errors are in parentheses. Superscripts a, b or c indicates significance at the 1%, 5% or 10% level, respectively.
Table 3: Analysis of portfolios constructed based on the asymptotic conditional crash likelihood measure

(a) portfolio L\textsuperscript{a} (bottom third)

<table>
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<tr>
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<td>0.09</td>
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<tr>
<td>(\beta)</td>
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<td>(\gamma)</td>
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<td>0.09</td>
<td>0.10</td>
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<td>0.22</td>
<td>-0.63</td>
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<tr>
<td>(\delta)</td>
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<td>0.14</td>
<td>0.09</td>
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<td>(\epsilon)</td>
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<td>0.10</td>
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<td>0.22</td>
<td>-0.63</td>
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<td>(\zeta)</td>
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(b) portfolio M\textsuperscript{a} (middle third)

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<td>-0.17</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.12</td>
<td>0.04</td>
<td>0.09</td>
<td>0.06</td>
<td>0.05</td>
<td>0.14</td>
<td>-0.03</td>
<td>0.13</td>
<td>-0.08</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.11</td>
<td>0.04</td>
<td>0.09</td>
<td>0.06</td>
<td>0.05</td>
<td>0.14</td>
<td>-0.03</td>
<td>0.13</td>
<td>-0.08</td>
</tr>
<tr>
<td>(\epsilon)</td>
<td>0.19</td>
<td>0.04</td>
<td>0.09</td>
<td>0.06</td>
<td>0.05</td>
<td>0.14</td>
<td>-0.03</td>
<td>0.13</td>
<td>-0.08</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>0.16</td>
<td>0.04</td>
<td>0.09</td>
<td>0.06</td>
<td>0.05</td>
<td>0.14</td>
<td>-0.03</td>
<td>0.13</td>
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(c) portfolio H\textsuperscript{a} (top third)

<table>
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<tr>
<th></th>
<th>(\alpha)</th>
<th>(r_m)</th>
<th>SMB</th>
<th>HML</th>
<th>UMD</th>
<th>NMP</th>
<th>LMP</th>
<th>NMP60</th>
<th>LMP60</th>
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<tbody>
<tr>
<td></td>
<td>0.61\textsuperscript{a}</td>
<td>0.70\textsuperscript{a}</td>
<td>-0.12\textsuperscript{a}</td>
<td>0.09</td>
<td>0.05</td>
<td>0.14\textsuperscript{a}</td>
<td>-0.03\textsuperscript{a}</td>
<td>0.12\textsuperscript{b}</td>
<td>-0.01\textsuperscript{a}</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.61</td>
<td>0.70</td>
<td>-0.12</td>
<td>0.09</td>
<td>0.05</td>
<td>0.14</td>
<td>-0.03</td>
<td>0.12</td>
<td>-0.01</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.19</td>
<td>0.04</td>
<td>0.09</td>
<td>0.06</td>
<td>0.05</td>
<td>0.14</td>
<td>-0.03</td>
<td>0.12</td>
<td>-0.01</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.19</td>
<td>0.04</td>
<td>0.09</td>
<td>0.06</td>
<td>0.05</td>
<td>0.14</td>
<td>-0.03</td>
<td>0.12</td>
<td>-0.01</td>
</tr>
<tr>
<td>(\epsilon)</td>
<td>0.19</td>
<td>0.04</td>
<td>0.09</td>
<td>0.06</td>
<td>0.05</td>
<td>0.14</td>
<td>-0.03</td>
<td>0.12</td>
<td>-0.01</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>0.16</td>
<td>0.04</td>
<td>0.09</td>
<td>0.06</td>
<td>0.05</td>
<td>0.14</td>
<td>-0.03</td>
<td>0.12</td>
<td>-0.01</td>
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(d) hedge portfolio (portfolio H\textsuperscript{a} - portfolio L\textsuperscript{a})

<table>
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<th></th>
<th>(\alpha)</th>
<th>(r_m)</th>
<th>SMB</th>
<th>HML</th>
<th>UMD</th>
<th>NMP</th>
<th>LMP</th>
<th>NMP60</th>
<th>LMP60</th>
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</thead>
<tbody>
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<td>0.12</td>
<td>0.07\textsuperscript{b}</td>
<td>-0.16\textsuperscript{b}</td>
<td>0.00</td>
<td>-0.06</td>
<td>-0.08</td>
<td>0.60\textsuperscript{b}</td>
<td>0.12</td>
<td>0.49\textsuperscript{a}</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.12</td>
<td>0.07</td>
<td>-0.16</td>
<td>0.00</td>
<td>-0.06</td>
<td>-0.08</td>
<td>0.60</td>
<td>0.12</td>
<td>0.49</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.17</td>
<td>0.07</td>
<td>-0.16</td>
<td>0.00</td>
<td>-0.06</td>
<td>-0.08</td>
<td>0.60</td>
<td>0.12</td>
<td>0.49</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.17</td>
<td>0.07</td>
<td>-0.16</td>
<td>0.00</td>
<td>-0.06</td>
<td>-0.08</td>
<td>0.60</td>
<td>0.12</td>
<td>0.49</td>
</tr>
<tr>
<td>(\epsilon)</td>
<td>0.17</td>
<td>0.07</td>
<td>-0.16</td>
<td>0.00</td>
<td>-0.06</td>
<td>-0.08</td>
<td>0.60</td>
<td>0.12</td>
<td>0.49</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>0.17</td>
<td>0.07</td>
<td>-0.16</td>
<td>0.00</td>
<td>-0.06</td>
<td>-0.08</td>
<td>0.60</td>
<td>0.12</td>
<td>0.49</td>
</tr>
</tbody>
</table>

This table present the regression results for different regressions of the monthly returns of the L\textsuperscript{a}, M\textsuperscript{a} and H\textsuperscript{a} portfolios on a constant, the excess market return and other factor portfolios. The L\textsuperscript{a}, M\textsuperscript{a} and H\textsuperscript{a} portfolios are value weighted portfolios with equal numbers of stocks, constructed based on the empirical measure for conditional crash likelihood (see Eq. (15)) for the different stocks, estimated over the preceding 120 months. As regressors we consider the excess market return \((r_m)\), the size factor (SMB), value factor (HML) and momentum factor (UMD) available on the website of French, and a coskewness factor (NMP) and cokurtosis factor (LMP) (both constructed as described in Appendix B, based on an estimation window of 120 months and of 60 months (labeled NMP60 and LMP60)). Newey-West standard errors are in parentheses. Superscripts a, b or c indicates significance at the 1%, 5% or 10% level, respectively.
This table presents the regression results for different regressions of the monthly returns of the $L^\nu$, $M^\nu$ and $H^\nu$ portfolios on a constant, the excess market return and other factor portfolios. The $L^\nu$, $M^\nu$ and $H^\nu$ portfolios are value weighted portfolios with equal numbers of stocks, constructed based on the empirical measure for conditional crash likelihood (see Eq. (16)) for the different stocks, estimated over the preceding 120 months. As regressors we consider the excess market return ($r_m$), the size factor (SMB), value factor (HML) and momentum factor (UMD) available on the website of French, and a coskewness factor (NMP) and cokurtosis factor (LMP) (both constructed as described in Appendix B, based on an estimation window of 120 months and of 60 months (labeled NMP60 and LMP60)). Newey-West standard errors are in parentheses. Superscripts a, b or c indicates significance at the 1%, 5% or 10% level, respectively.
Table 5: Cross sectional tests with and without crash risk on single sorted portfolios

<table>
<thead>
<tr>
<th>(a) Industry portfolios</th>
<th>(b) Size portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td>ζ&lt;sub&gt;d&lt;/sub&gt;</td>
<td>ζ&lt;sub&gt;d'&lt;/sub&gt;</td>
</tr>
<tr>
<td>0.67&lt;sup&gt;a&lt;/sup&gt; (0.25)</td>
<td>0.66&lt;sup&gt;a&lt;/sup&gt; (0.25)</td>
</tr>
<tr>
<td>0.57 (0.55)</td>
<td>0.76&lt;sup&gt;a&lt;/sup&gt; (0.26)</td>
</tr>
<tr>
<td>1</td>
<td>0.27&lt;sup&gt;c&lt;/sup&gt; (0.16)</td>
</tr>
<tr>
<td>2</td>
<td>-0.08 (0.18)</td>
</tr>
<tr>
<td>3</td>
<td>-0.13 (0.11)</td>
</tr>
<tr>
<td>4</td>
<td>0.18 (0.23)</td>
</tr>
<tr>
<td>5</td>
<td>-0.30 (0.25)</td>
</tr>
<tr>
<td>6</td>
<td>0.05 (0.20)</td>
</tr>
<tr>
<td>7</td>
<td>0.00 (0.13)</td>
</tr>
<tr>
<td>8</td>
<td>0.10 (0.18)</td>
</tr>
<tr>
<td>9</td>
<td>0.23 (0.18)</td>
</tr>
<tr>
<td>10</td>
<td>0.03 (0.10)</td>
</tr>
</tbody>
</table>

| TJ<sub>F</sub> | 9.31 [0.41] | 6.79 [0.56] | 8.51 [0.39] | 9.03 [0.43] | 6.22 [0.62] | 7.37 [0.50] |

<table>
<thead>
<tr>
<th>(c) Value portfolios</th>
<th>(d) Momentum portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td>ζ&lt;sub&gt;d'&lt;/sub&gt;</td>
<td>ζ&lt;sub&gt;c'&lt;/sub&gt;</td>
</tr>
<tr>
<td>0.80&lt;sup&gt;a&lt;/sup&gt; (0.25)</td>
<td>0.78&lt;sup&gt;a&lt;/sup&gt; (0.24)</td>
</tr>
<tr>
<td>0.84 (0.80)</td>
<td>1.70&lt;sup&gt;b&lt;/sup&gt; (0.70)</td>
</tr>
<tr>
<td>1</td>
<td>-0.51&lt;sup&gt;b&lt;/sup&gt; (0.20)</td>
</tr>
<tr>
<td>2</td>
<td>-0.14 (0.09)</td>
</tr>
<tr>
<td>3</td>
<td>-0.10 (0.07)</td>
</tr>
<tr>
<td>4</td>
<td>0.00 (0.09)</td>
</tr>
<tr>
<td>5</td>
<td>0.04 (0.09)</td>
</tr>
<tr>
<td>6</td>
<td>0.07 (0.07)</td>
</tr>
<tr>
<td>7</td>
<td>0.20&lt;sup&gt;b&lt;/sup&gt; (0.10)</td>
</tr>
<tr>
<td>8</td>
<td>0.15 (0.10)</td>
</tr>
<tr>
<td>9</td>
<td>0.21&lt;sup&gt;b&lt;/sup&gt; (0.10)</td>
</tr>
<tr>
<td>10</td>
<td>0.29&lt;sup&gt;c&lt;/sup&gt; (0.17)</td>
</tr>
</tbody>
</table>

| TJ<sub>F</sub> | 8.39 [0.50] | 7.53 [0.48] | 5.91 [0.66] | 21.28<sup>b</sup> [0.01] | 11.38 [0.18] | 9.86 [0.27] |

This table reports the results of cross-sectional tests of three asset pricing models on four sets of portfolios: portfolios based on industry (panel a), based on size (panel b), based on book-to-market ratios (panel c) and based on momentum. We consider the CAPM, the CAPM extended with the hedge portfolio based on the empirical measure for conditional crash likelihood and the CAPM extended with the H portfolios based on the asymptotic measure for conditional crash likelihood. We estimate the sensitivities and risk premia in a GMM framework. For the industry, size and value portfolios we use observations from June 1974 - November 2003. For the momentum portfolios we use observations from June 1974 - December 2002. We report premia on market diffusion risk (ζ), on the empirically based hedge portfolio (ζ<sub>e'</sub>), and on the asymptotically based H<sub>exta</sub> portfolio (ζ<sub>s'</sub>). After the numbers 1 to 10 we report the pricing errors for the different portfolios. The industry portfolios are ordered as Consumer Non-Durables (1), Consumer Durables (2), Manufacturing (3), Energy (4), High Tech (5), Telecom (6), Shops (7), Health (8), Utilities (9) and Others (10). The size portfolios are ordered from small (1) to big (10). The value portfolios are ordered from low (1) to high (10). The momentum portfolios are ordered from loser (1) to winner (10). After each estimate the Newey and West (1987) consistent standard error is reported in parentheses. TJ<sub>F</sub> reports the value of the TJ<sub>F</sub> statistic, with the p-value for an insignificant TJ<sub>F</sub> statistic based on a χ² distribution in brackets. Superscripts a, b or c indicates significance at the 1%, 5% or 10% level, respectively.
This table reports the results for a cross-sectional test of three asset pricing models on the entire cross section of stocks. The test is conducted in a Fama and MacBeth (1973)-framework, with an estimation window of 60 months. The returns of month 61 are then used to estimate the risk premia and to construct pricing errors. We consider all stocks in the CRSP database from June 1974 to November 2003. To be included in the analysis a stock should have at least a complete series 61 observations. We report the time series averages of the risk premia. For each month, we calculate the cross-sectional average pricing error, and we report the time series average of this series ($\bar{\alpha}$). We report the premium on market diffusion risk ($\zeta_d$), the premium on the empirically based hedge portfolio ($\zeta_{emp}$), and the premium on the asymptotically based $H^{extra}$ portfolio ($\zeta_a$). We also report estimates of the standard errors that are only based on the time series of the premia and pricing errors (column se) and standard error estimates that include a correction for the fact that the risk sensitivities are estimated based on Shanken (1992) (column se+). We calculate the additive correction as the variance of the risk factors divided by the number of observations, and the multiplicative correction as $\zeta'\Sigma_f^{-1}\zeta$, where $\zeta$ denotes the vector of estimated risk premia, and $\Sigma_f$ is the factor variance matrix (see Cochrane, 2001, Ch. 12, for a discussion). All standard errors are based on a Newey and West (1987) correction for autocorrelation and heteroskedasticity.
Table 7: Analysis of the market portfolio and hedge portfolios

<table>
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<tr>
<th></th>
<th>$r_m$</th>
<th>SMB</th>
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<th>UMD</th>
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<th>NMP60</th>
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<th>LMP60</th>
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<tbody>
<tr>
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<td>0.62^a</td>
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<td>0.04</td>
<td>0.27^c</td>
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<td>−0.14</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(0.16)</td>
<td>(0.18)</td>
<td>(0.23)</td>
<td>(0.14)</td>
<td>(0.15)</td>
<td>(0.12)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>$r_m$</td>
<td>0.16^a</td>
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<td>−0.13</td>
<td>0.07</td>
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<td>(0.05)</td>
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<td>(0.03)</td>
<td>(0.04)</td>
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<tr>
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</table>

This table presents the regression results for the monthly return series (in %) of the excess market index and the different hedge portfolio over the period June 1974 - November 2003. We consider the market return $r_m$, and the hedge portfolios based on the size effect (SMB), the value effect (HML), the momentum effect (UMD), coskewness (NMP, NMP60) and cokurtosis (LMP and LMP60). The SMB, HML and UMD return series are available on French’s web site. To construct hedge portfolios for coskewness and cokurtosis we first construct market value weighted L, M and H portfolios for the two measures. The measure for coskewness is given in Eq. (B1), and is estimated over 120 months (NMP) or 60 months (NMP60). The measure for cokurtosis, given in Eq. (B2), is also estimated over 120 months (PML) or 60 months (PML60). The coskewness hedge portfolios are constructed as a long position in L portfolio and a short position in the H portfolio. The cokurtosis hedge portfolios are constructed with a long position in the H portfolio and a short position in the L portfolio. We regress the hedge portfolios on a constant, the excess market return and other hedge portfolios. Each columns presents the results for a different hedge portfolio. Newey and West (1987) standard errors are in parentheses. Superscripts a, b or c indicates significance at the 1%, 5% or 10% level, respectively.
This figure shows the one third and two thirds quantiles of the distribution of the three measures for the probability that a stock crashes given that the market crashes. The empirical measure (panel a) is calculated as the conditional probability of a stock return below the 5%-quantile, given that the market return falls below the 5%-quantile based on the empirical copula minus that conditional probability according to the Gaussian copula as in Eq. (12). The asymptotic measure (panel b) is calculated as the tail dependence coefficient that is implied by the Student’s $t$ copula, using Eq. (15). The degrees of freedom measure (panel c) is the transformation in Eq. (16) of the degrees of freedom parameters of the Student’s $t$ copula. We calculate the measure for each stock in each month, if the return history is long enough. The empirical copula and the parameter estimates for the Gaussian and Student’s $t$ copulas are based on the previous 120 monthly excess stock returns and excess market returns, using the IFM method of Joe (1997). Based on the cross section of stocks in each month, we calculate the values of the measures below which one third and two thirds of the stocks in that month lie. The solid (dashed) line corresponds with the one third (two thirds) quantile.